

# Mathematica 11.3 Integration Test Results

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Test results for the 705 problems in "Timofeev Problems.m"

Problem 3: Result more than twice size of optimal antiderivative.

$$\int \sec[2ax] dx$$

Optimal (type 3, 13 leaves, 1 step):

$$\frac{\text{ArcTanh}[\text{Sin}[2ax]]}{2a}$$

Result (type 3, 37 leaves):

$$-\frac{\text{Log}[\text{Cos}[ax] - \text{Sin}[ax]]}{2a} + \frac{\text{Log}[\text{Cos}[ax] + \text{Sin}[ax]]}{2a}$$

Problem 4: Result more than twice size of optimal antiderivative.

$$\int \frac{1}{4} \csc\left[\frac{x}{3}\right] dx$$

Optimal (type 3, 11 leaves, 2 steps):

$$-\frac{3}{4} \text{ArcTanh}\left[\text{Cos}\left[\frac{x}{3}\right]\right]$$

Result (type 3, 23 leaves):

$$\frac{1}{4} \left( -3 \text{Log}\left[\text{Cos}\left[\frac{x}{6}\right]\right] + 3 \text{Log}\left[\text{Sin}\left[\frac{x}{6}\right]\right] \right)$$

Problem 5: Result more than twice size of optimal antiderivative.

$$\int -\sec\left[\frac{\pi}{4} + 2x\right] dx$$

Optimal (type 3, 15 leaves, 1 step):

$$-\frac{1}{2} \text{ArcTanh}\left[\text{Sin}\left[\frac{\pi}{4} + 2x\right]\right]$$

Result (type 3, 55 leaves):

$$\frac{1}{2} \text{Log}\left[\text{Cos}\left[\frac{1}{8}(\pi + 8x)\right] - \text{Sin}\left[\frac{1}{8}(\pi + 8x)\right]\right] - \frac{1}{2} \text{Log}\left[\text{Cos}\left[\frac{1}{8}(\pi + 8x)\right] + \text{Sin}\left[\frac{1}{8}(\pi + 8x)\right]\right]$$

**Problem 19: Result more than twice size of optimal antiderivative.**

$$\int \frac{e^x}{-1 + e^{2x}} dx$$

Optimal (type 3, 6 leaves, 2 steps):

$$-\text{ArcTanh}[e^x]$$

Result (type 3, 23 leaves):

$$\frac{1}{2} \text{Log}[1 - e^x] - \frac{1}{2} \text{Log}[1 + e^x]$$

**Problem 35: Result more than twice size of optimal antiderivative.**

$$\int \text{Cot}[x]^3 \text{Csc}[x] dx$$

Optimal (type 3, 11 leaves, 2 steps):

$$\text{Csc}[x] - \frac{\text{Csc}[x]^3}{3}$$

Result (type 3, 57 leaves):

$$\frac{5}{12} \text{Cot}\left[\frac{x}{2}\right] - \frac{1}{24} \text{Cot}\left[\frac{x}{2}\right] \text{Csc}\left[\frac{x}{2}\right]^2 + \frac{5}{12} \text{Tan}\left[\frac{x}{2}\right] - \frac{1}{24} \text{Sec}\left[\frac{x}{2}\right]^2 \text{Tan}\left[\frac{x}{2}\right]$$

**Problem 41: Result more than twice size of optimal antiderivative.**

$$\int \frac{\text{Sin}[x]}{1 + \text{Sin}[x]} dx$$

Optimal (type 3, 11 leaves, 2 steps):

$$x + \frac{\text{Cos}[x]}{1 + \text{Sin}[x]}$$

Result (type 3, 25 leaves):

$$x - \frac{2 \operatorname{Sin}\left[\frac{x}{2}\right]}{\operatorname{Cos}\left[\frac{x}{2}\right] + \operatorname{Sin}\left[\frac{x}{2}\right]}$$

Problem 51: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{x \sqrt{-a^2 + x^2}} dx$$

Optimal (type 3, 22 leaves, 3 steps):

$$\frac{\operatorname{ArcTan}\left[\frac{\sqrt{-a^2 + x^2}}{a}\right]}{a}$$

Result (type 3, 35 leaves):

$$-\frac{i \operatorname{Log}\left[-\frac{2 i a}{x} + \frac{2 \sqrt{-a^2 + x^2}}{x}\right]}{a}$$

Problem 56: Result more than twice size of optimal antiderivative.

$$\int \frac{1}{\sqrt{x - x^2}} dx$$

Optimal (type 3, 8 leaves, 2 steps):

$$-\operatorname{ArcSin}[1 - 2x]$$

Result (type 3, 38 leaves):

$$\frac{2 \sqrt{-1 + x} \sqrt{x} \operatorname{Log}\left[\sqrt{-1 + x} + \sqrt{x}\right]}{\sqrt{-(-1 + x)x}}$$

Problem 63: Result more than twice size of optimal antiderivative.

$$\int \frac{1 + \operatorname{Tan}[x]^2}{1 - \operatorname{Tan}[x]^2} dx$$

Optimal (type 3, 11 leaves, 2 steps):

$$\frac{1}{2} \text{ArcTanh}[2 \text{Cos}[x] \text{Sin}[x]]$$

Result (type 3, 23 leaves):

$$-\frac{1}{2} \text{Log}[\text{Cos}[x] - \text{Sin}[x]] + \frac{1}{2} \text{Log}[\text{Cos}[x] + \text{Sin}[x]]$$

**Problem 64: Result more than twice size of optimal antiderivative.**

$$\int (a^2 - 4 \text{Cos}[x]^2)^{3/4} \text{Sin}[2x] \, dx$$

Optimal (type 3, 18 leaves, 3 steps):

$$\frac{1}{7} (a^2 - 4 \text{Cos}[x]^2)^{7/4}$$

Result (type 3, 127 leaves):

$$\frac{1}{7 (-2 + a^2 - 2 \text{Cos}[2x])^{1/4}} \left( 6 - 4a^2 + a^4 - 4 \left( \frac{-2 + a^2 - 2 \text{Cos}[2x]}{-2 + a^2} \right)^{1/4} + 4a^2 \left( \frac{-2 + a^2 - 2 \text{Cos}[2x]}{-2 + a^2} \right)^{1/4} - a^4 \left( \frac{-2 + a^2 - 2 \text{Cos}[2x]}{-2 + a^2} \right)^{1/4} - 4(-2 + a^2) \text{Cos}[2x] + 2 \text{Cos}[4x] \right)$$

**Problem 65: Result more than twice size of optimal antiderivative.**

$$\int \frac{\text{Sin}[2x]}{(a^2 - 4 \text{Sin}[x]^2)^{1/3}} \, dx$$

Optimal (type 3, 18 leaves, 3 steps):

$$-\frac{3}{8} (a^2 - 4 \text{Sin}[x]^2)^{2/3}$$

Result (type 3, 67 leaves):

$$-\frac{3 (-2 + a^2 + 2 \text{Cos}[2x])^{2/3} \left( -1 + \left( \frac{-2 + a^2 + 2 \text{Cos}[2x]}{-2 + a^2} \right)^{2/3} \right)}{8 \left( \frac{-2 + a^2 + 2 \text{Cos}[2x]}{-2 + a^2} \right)^{2/3}}$$

**Problem 76: Result more than twice size of optimal antiderivative.**

$$\int \text{Csc}[x]^5 \, dx$$

Optimal (type 3, 26 leaves, 3 steps):

$$-\frac{3}{8} \operatorname{ArcTanh}[\cos[x]] - \frac{3}{8} \cot[x] \operatorname{Csc}[x] - \frac{1}{4} \cot[x] \operatorname{Csc}[x]^3$$

Result (type 3, 71 leaves):

$$-\frac{3}{32} \operatorname{Csc}\left[\frac{x}{2}\right]^2 - \frac{1}{64} \operatorname{Csc}\left[\frac{x}{2}\right]^4 - \frac{3}{8} \operatorname{Log}\left[\cos\left[\frac{x}{2}\right]\right] + \frac{3}{8} \operatorname{Log}\left[\sin\left[\frac{x}{2}\right]\right] + \frac{3}{32} \operatorname{Sec}\left[\frac{x}{2}\right]^2 + \frac{1}{64} \operatorname{Sec}\left[\frac{x}{2}\right]^4$$

**Problem 99: Result more than twice size of optimal antiderivative.**

$$\int \frac{-5 + 2x^2}{6 - 5x^2 + x^4} dx$$

Optimal (type 3, 31 leaves, 3 steps):

$$-\frac{\operatorname{ArcTanh}\left[\frac{x}{\sqrt{2}}\right]}{\sqrt{2}} - \frac{\operatorname{ArcTanh}\left[\frac{x}{\sqrt{3}}\right]}{\sqrt{3}}$$

Result (type 3, 69 leaves):

$$\frac{1}{12} \left( 3\sqrt{2} \operatorname{Log}[\sqrt{2} - x] + 2\sqrt{3} \operatorname{Log}[\sqrt{3} - x] - 3\sqrt{2} \operatorname{Log}[\sqrt{2} + x] - 2\sqrt{3} \operatorname{Log}[\sqrt{3} + x] \right)$$

**Problem 113: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{1}{1 + x^2 + x^4} dx$$

Optimal (type 3, 67 leaves, 9 steps):

$$-\frac{\operatorname{ArcTan}\left[\frac{1-2x}{\sqrt{3}}\right]}{2\sqrt{3}} + \frac{\operatorname{ArcTan}\left[\frac{1+2x}{\sqrt{3}}\right]}{2\sqrt{3}} - \frac{1}{4} \operatorname{Log}[1 - x + x^2] + \frac{1}{4} \operatorname{Log}[1 + x + x^2]$$

Result (type 3, 73 leaves):

$$\frac{i \left( \sqrt{1 - i\sqrt{3}} \operatorname{ArcTan}\left[\frac{1}{2}(-i + \sqrt{3})x\right] - \sqrt{1 + i\sqrt{3}} \operatorname{ArcTan}\left[\frac{1}{2}(i + \sqrt{3})x\right] \right)}{\sqrt{6}}$$

**Problem 193:** Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int (b_1 + c_1 x) (a + 2 b x + c x^2)^n dx$$

Optimal (type 5, 159 leaves, 2 steps):

$$\frac{c_1 (a + 2 b x + c x^2)^{1+n}}{2 c (1+n)} - \frac{2^n (b_1 c - b c_1) \left( -\frac{b - \sqrt{b^2 - a c} + c x}{\sqrt{b^2 - a c}} \right)^{-1-n} (a + 2 b x + c x^2)^{1+n} \text{Hypergeometric2F1} \left[ -n, 1+n, 2+n, \frac{b + \sqrt{b^2 - a c} + c x}{2 \sqrt{b^2 - a c}} \right]}{c \sqrt{b^2 - a c} (1+n)}$$

Result (type 6, 471 leaves):

$$\frac{1}{2} \left( b - \sqrt{b^2 - a c} + c x \right) (a + x (2 b + c x))^n$$

$$\left( \left( 3 \left( b + \sqrt{b^2 - a c} \right) c_1 x^2 \left( a + \left( b - \sqrt{b^2 - a c} \right) x \right)^2 \text{AppellF1} \left[ 2, -n, -n, 3, -\frac{c x}{b + \sqrt{b^2 - a c}}, \frac{c x}{-b + \sqrt{b^2 - a c}} \right] \right) / \right.$$

$$\left( \left( -b + \sqrt{b^2 - a c} \right) \left( b + \sqrt{b^2 - a c} + c x \right) (a + x (2 b + c x)) \left( -3 a \text{AppellF1} \left[ 2, -n, -n, 3, -\frac{c x}{b + \sqrt{b^2 - a c}}, \frac{c x}{-b + \sqrt{b^2 - a c}} \right] + \right.$$

$$\left. n x \left( \left( -b + \sqrt{b^2 - a c} \right) \text{AppellF1} \left[ 3, 1-n, -n, 4, -\frac{c x}{b + \sqrt{b^2 - a c}}, \frac{c x}{-b + \sqrt{b^2 - a c}} \right] - \left( b + \sqrt{b^2 - a c} \right) \text{AppellF1} \left[ 3, -n, 1-n, 4, \right. \right.$$

$$\left. \left. -\frac{c x}{b + \sqrt{b^2 - a c}}, \frac{c x}{-b + \sqrt{b^2 - a c}} \right] \right) \right) + \frac{2^{1+n} b_1 \left( \frac{b + \sqrt{b^2 - a c} + c x}{\sqrt{b^2 - a c}} \right)^{-n} \text{Hypergeometric2F1} \left[ -n, 1+n, 2+n, \frac{-b + \sqrt{b^2 - a c} - c x}{2 \sqrt{b^2 - a c}} \right]}{c (1+n)} \right)$$

**Problem 198:** Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int (b_1 + c_1 x) (a + 2 b x + c x^2)^{-n} dx$$

Optimal (type 5, 169 leaves, 2 steps):

$$\frac{c_1 (a + 2bx + cx^2)^{1-n}}{2c(1-n)} - \frac{2^{-n} (b_1c - bc_1) \left( -\frac{b - \sqrt{b^2 - ac} + cx}{\sqrt{b^2 - ac}} \right)^{-1+n} (a + 2bx + cx^2)^{1-n} \text{Hypergeometric2F1} \left[ 1 - n, n, 2 - n, \frac{b + \sqrt{b^2 - ac} + cx}{2\sqrt{b^2 - ac}} \right]}{c\sqrt{b^2 - ac}(1-n)}$$

Result (type 6, 374 leaves):

$$\frac{1}{2} (a + x(2b + cx))^{-n} \left( - \left( \left( 3ac_1x^2 \text{AppellF1} \left[ 2, n, n, 3, -\frac{cx}{b + \sqrt{b^2 - ac}}, \frac{cx}{-b + \sqrt{b^2 - ac}} \right] \right) / \left( -3a \text{AppellF1} \left[ 2, n, n, 3, -\frac{cx}{b + \sqrt{b^2 - ac}}, \frac{cx}{-b + \sqrt{b^2 - ac}} \right] \right) + \right. \\ \left. nx \left( \left( (b + \sqrt{b^2 - ac}) \text{AppellF1} \left[ 3, n, 1 + n, 4, -\frac{cx}{b + \sqrt{b^2 - ac}}, \frac{cx}{-b + \sqrt{b^2 - ac}} \right] + \right. \right. \right. \\ \left. \left. \left( (b - \sqrt{b^2 - ac}) \text{AppellF1} \left[ 3, 1 + n, n, 4, -\frac{cx}{b + \sqrt{b^2 - ac}}, \frac{cx}{-b + \sqrt{b^2 - ac}} \right] \right) \right) \right) - \\ \left. \frac{2^{1-n} b_1 (b - \sqrt{b^2 - ac} + cx) \left( \frac{b + \sqrt{b^2 - ac} + cx}{\sqrt{b^2 - ac}} \right)^n \text{Hypergeometric2F1} \left[ 1 - n, n, 2 - n, \frac{-b + \sqrt{b^2 - ac} - cx}{2\sqrt{b^2 - ac}} \right]}{c(-1+n)} \right)$$

Problem 217: Result unnecessarily involves higher level functions.

$$\int \frac{1}{(-1+x)^{2/3} x^5} dx$$

Optimal (type 3, 104 leaves, 8 steps):

$$\frac{(-1+x)^{1/3}}{4x^4} + \frac{11(-1+x)^{1/3}}{36x^3} + \frac{11(-1+x)^{1/3}}{27x^2} + \frac{55(-1+x)^{1/3}}{81x} - \frac{110 \text{ArcTan} \left[ \frac{1-2(-1+x)^{1/3}}{\sqrt{3}} \right]}{81\sqrt{3}} + \frac{55}{81} \text{Log} \left[ 1 + (-1+x)^{1/3} \right] - \frac{55 \text{Log}[x]}{243}$$

Result (type 5, 63 leaves):

$$\frac{-81 - 18x - 33x^2 - 88x^3 + 220x^4 - 220 \left( \frac{-1+x}{x} \right)^{2/3} x^4 \text{Hypergeometric2F1} \left[ \frac{2}{3}, \frac{2}{3}, \frac{5}{3}, \frac{1}{x} \right]}{324(-1+x)^{2/3} x^4}$$

### Problem 221: Result unnecessarily involves higher level functions.

$$\int \frac{x^2 \sqrt{1+x} (1-x^2)^{1/4}}{\sqrt{1-x} (\sqrt{1-x} - \sqrt{1+x})} dx$$

Optimal (type 3, 304 leaves, 33 steps):

$$\frac{5}{16} (1-x)^{3/4} (1+x)^{1/4} - \frac{1}{16} (1-x)^{1/4} (1+x)^{3/4} + \frac{1}{24} (1-x)^{5/4} (1+x)^{3/4} + \frac{7(1-x^2)^{5/4}}{24\sqrt{1-x}} + \frac{x(1-x^2)^{5/4}}{6\sqrt{1-x}} + \frac{1}{6} \sqrt{1+x} (1-x^2)^{5/4} -$$

$$\frac{3 \operatorname{ArcTan}\left[1 - \frac{\sqrt{2}(1-x)^{1/4}}{(1+x)^{1/4}}\right]}{8\sqrt{2}} + \frac{3 \operatorname{ArcTan}\left[1 + \frac{\sqrt{2}(1-x)^{1/4}}{(1+x)^{1/4}}\right]}{8\sqrt{2}} + \frac{\operatorname{Log}\left[1 + \frac{\sqrt{1-x}}{\sqrt{1+x}} - \frac{\sqrt{2}(1-x)^{1/4}}{(1+x)^{1/4}}\right]}{8\sqrt{2}} - \frac{\operatorname{Log}\left[1 + \frac{\sqrt{1-x}}{\sqrt{1+x}} + \frac{\sqrt{2}(1-x)^{1/4}}{(1+x)^{1/4}}\right]}{8\sqrt{2}}$$

Result (type 5, 165 leaves):

$$-\frac{1}{48} \sqrt{1+x} (1-x^2)^{1/4} \left( -7 + 2x + 8x^2 - \frac{\sqrt{1-x^2} (29 + 22x + 8x^2)}{1+x} \right) +$$

$$\frac{(-2(-1+x) - (-1+x)^2)^{1/4} \operatorname{Hypergeometric2F1}\left[\frac{1}{4}, \frac{1}{4}, \frac{5}{4}, \frac{1-x}{2}\right]}{8 \times 2^{1/4} (1+x)^{1/4}} + \frac{5(-2(-1+x) - (-1+x)^2)^{3/4} \operatorname{Hypergeometric2F1}\left[\frac{3}{4}, \frac{3}{4}, \frac{7}{4}, \frac{1-x}{2}\right]}{24 \times 2^{3/4} (1+x)^{3/4}}$$

### Problem 222: Result unnecessarily involves higher level functions.

$$\int \frac{\sqrt{1-x} x (1+x)^{2/3}}{-(1-x)^{5/6} (1+x)^{1/3} + (1-x)^{2/3} \sqrt{1+x}} dx$$

Optimal (type 3, 292 leaves, ? steps):

$$-\frac{1}{12} (1-3x) (1-x)^{2/3} (1+x)^{1/3} + \frac{1}{4} \sqrt{1-x} x \sqrt{1+x} - \frac{1}{4} (1-x) (3+x) +$$

$$\frac{1}{12} (1-x)^{1/3} (1+x)^{2/3} (1+3x) + \frac{1}{12} (1-x)^{1/6} (1+x)^{5/6} (2+3x) - \frac{1}{12} (1-x)^{5/6} (1+x)^{1/6} (10+3x) +$$

$$\frac{1}{6} \operatorname{ArcTan}\left[\frac{(1+x)^{1/6}}{(1-x)^{1/6}}\right] - \frac{4 \operatorname{ArcTan}\left[\frac{(1-x)^{1/3} - 2(1+x)^{1/3}}{\sqrt{3}(1-x)^{1/3}}\right]}{3\sqrt{3}} - \frac{5}{6} \operatorname{ArcTan}\left[\frac{(1-x)^{1/3} - (1+x)^{1/3}}{(1-x)^{1/6} (1+x)^{1/6}}\right] + \frac{\operatorname{ArcTanh}\left[\frac{\sqrt{3}(1-x)^{1/6} (1+x)^{1/6}}{(1-x)^{1/3} + (1+x)^{1/3}}\right]}{6\sqrt{3}}$$

Result (type 5, 391 leaves):



$$\begin{aligned}
& - \frac{2^{2/3} \left( -2 (-1+x) - (-1+x)^2 \right)^{1/3} \text{Hypergeometric2F1} \left[ \frac{1}{3}, \frac{1}{3}, \frac{4}{3}, \frac{1-x}{2} \right]}{3 (1+x)^{1/3}} - \\
& \frac{1}{12} (1+x)^{1/3} \left( (1-3x) (1-x)^{2/3} - \frac{3 (1-x)^{1/3} x (2+x)}{(1-x^2)^{1/3}} - 3 (1-x)^{1/3} x (1-x^2)^{1/6} - (1+3x) (1-x^2)^{1/3} - \frac{(2+3x) \sqrt{1-x^2}}{(1-x)^{1/3}} + \frac{(10+3x) (1-x^2)^{5/6}}{1+x} \right) - \\
& 4 \times 2^{2/3} \text{Hypergeometric2F1} \left[ \frac{1}{3}, \frac{1}{3}, \frac{4}{3}, \frac{1+x}{2} \right] - \frac{7 \left( -2 (-1+x) - (-1+x)^2 \right)^{5/6} \text{Hypergeometric2F1} \left[ \frac{5}{6}, \frac{5}{6}, \frac{11}{6}, \frac{1-x}{2} \right]}{30 \times 2^{5/6} (1+x)^{5/6}} + \\
& \frac{(1+x)^{1/3} \sqrt{2(1+x) - (1+x)^2} \text{Hypergeometric2F1} \left[ \frac{5}{6}, \frac{5}{6}, \frac{11}{6}, \frac{1+x}{2} \right]}{6 \times 2^{5/6} \sqrt{1-x}} + \frac{(1-x)^{1/3} \sqrt{-1+x} (1+x)^{5/6} \text{Log} [\sqrt{-1+x} + \sqrt{1+x}]}{2 \left( 2(1+x) - (1+x)^2 \right)^{5/6}}
\end{aligned}$$

**Problem 226: Result unnecessarily involves higher level functions.**

$$\int \frac{1}{\left( (-1+x)^2 (1+x) \right)^{1/3}} dx$$

Optimal (type 3, 67 leaves, ? steps):

$$\sqrt{3} \text{ArcTan} \left[ \frac{1 + \frac{2(-1+x)}{\left( (-1+x)^2 (1+x) \right)^{1/3}}}{\sqrt{3}} \right] - \frac{1}{2} \text{Log} [1+x] - \frac{3}{2} \text{Log} \left[ 1 - \frac{-1+x}{\left( (-1+x)^2 (1+x) \right)^{1/3}} \right]$$

Result (type 5, 49 leaves):

$$\frac{3 (-1+x) (1+x)^{1/3} \text{Hypergeometric2F1} \left[ \frac{1}{3}, \frac{1}{3}, \frac{4}{3}, \frac{1-x}{2} \right]}{2^{1/3} \left( (-1+x)^2 (1+x) \right)^{1/3}}$$

**Problem 228: Result unnecessarily involves higher level functions.**

$$\int \frac{\left( (-1+x)^2 (1+x) \right)^{1/3}}{x^2} dx$$

Optimal (type 3, 150 leaves, ? steps):

$$-\frac{\left((-1+x)^2(1+x)\right)^{1/3}}{x} - \frac{\text{ArcTan}\left[\frac{1-\frac{2(-1+x)}{\left((-1+x)^2(1+x)\right)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3}} - \sqrt{3} \text{ArcTan}\left[\frac{1+\frac{2(-1+x)}{\left((-1+x)^2(1+x)\right)^{1/3}}}{\sqrt{3}}\right] +$$

$$\frac{\text{Log}[x]}{6} - \frac{2}{3} \text{Log}[1+x] - \frac{3}{2} \text{Log}\left[1 - \frac{-1+x}{\left((-1+x)^2(1+x)\right)^{1/3}}\right] - \frac{1}{2} \text{Log}\left[1 + \frac{-1+x}{\left((-1+x)^2(1+x)\right)^{1/3}}\right]$$

Result (type 6, 145 leaves):

$$\frac{1}{2} \left( (-1+x)^2(1+x) \right)^{1/3} \left( -\frac{2}{x} - \left( 4 \times \text{AppellF1}\left[1, \frac{1}{3}, \frac{2}{3}, 2, \frac{1}{x}, -\frac{1}{x}\right] \right) \right) /$$

$$\left( (-1+x)(1+x) \left( 6 \times \text{AppellF1}\left[1, \frac{1}{3}, \frac{2}{3}, 2, \frac{1}{x}, -\frac{1}{x}\right] - 2 \text{AppellF1}\left[2, \frac{1}{3}, \frac{5}{3}, 3, \frac{1}{x}, -\frac{1}{x}\right] + \text{AppellF1}\left[2, \frac{4}{3}, \frac{2}{3}, 3, \frac{1}{x}, -\frac{1}{x}\right] \right) \right) -$$

$$\frac{3 \times 2^{2/3} \text{Hypergeometric2F1}\left[\frac{1}{3}, \frac{1}{3}, \frac{4}{3}, \frac{1+x}{2}\right]}{(1-x)^{2/3}}$$

Problem 232: Result unnecessarily involves higher level functions.

$$\int \frac{1}{(9+3x-5x^2+x^3)^{1/3}} dx$$

Optimal (type 3, 75 leaves, ? steps):

$$\sqrt{3} \text{ArcTan}\left[\frac{1+\frac{2(-3+x)}{(9+3x-5x^2+x^3)^{1/3}}}{\sqrt{3}}\right] - \frac{1}{2} \text{Log}[1+x] - \frac{3}{2} \text{Log}\left[1 - \frac{-3+x}{(9+3x-5x^2+x^3)^{1/3}}\right]$$

Result (type 5, 49 leaves):

$$\frac{3(-3+x)(1+x)^{1/3} \text{Hypergeometric2F1}\left[\frac{1}{3}, \frac{1}{3}, \frac{4}{3}, \frac{3-x}{4}\right]}{2^{2/3} \left((-3+x)^2(1+x)\right)^{1/3}}$$

Problem 245: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \frac{x}{(4+x+x^2)\sqrt{5+4x+4x^2}} dx$$

Optimal (type 3, 63 leaves, 5 steps):

$$\frac{\text{ArcTan}\left[\frac{\sqrt{5+4x+4x^2}}{\sqrt{11}}\right]}{\sqrt{11}} - \frac{\text{ArcTanh}\left[\frac{\sqrt{\frac{11}{15}(1+2x)}}{\sqrt{5+4x+4x^2}}\right]}{\sqrt{165}}$$

Result (type 3, 426 leaves):

$$\begin{aligned} & \frac{(\mathbf{i} + \sqrt{15}) \text{ArcTan}\left[\frac{-4\sqrt{15} - \sqrt{15}x - \sqrt{15}x^2 - 4\sqrt{11}\sqrt{5+4x+4x^2}}{16+15x+15x^2}\right]}{2\sqrt{165}} + \frac{(-\mathbf{i} + \sqrt{15}) \text{ArcTan}\left[\frac{4\sqrt{15} + \sqrt{15}x + \sqrt{15}x^2 - 4\sqrt{11}\sqrt{5+4x+4x^2}}{16+15x+15x^2}\right]}{2\sqrt{165}} + \\ & \frac{\mathbf{i}(-\mathbf{i} + \sqrt{15}) \text{Log}\left[\left(-\mathbf{i} + \sqrt{15} - 2\mathbf{i}x\right)^2 \left(\mathbf{i} + \sqrt{15} + 2\mathbf{i}x\right)^2\right]}{4\sqrt{165}} + \frac{\mathbf{i}(\mathbf{i} + \sqrt{15}) \text{Log}\left[\left(-\mathbf{i} + \sqrt{15} - 2\mathbf{i}x\right)^2 \left(\mathbf{i} + \sqrt{15} + 2\mathbf{i}x\right)^2\right]}{4\sqrt{165}} - \\ & \frac{\mathbf{i}(\mathbf{i} + \sqrt{15}) \text{Log}\left[\left(4+x+x^2\right)\left(43+52x+52x^2 - \sqrt{165}\sqrt{5+4x+4x^2} - 2\sqrt{165}x\sqrt{5+4x+4x^2}\right)\right]}{4\sqrt{165}} - \\ & \frac{\mathbf{i}(-\mathbf{i} + \sqrt{15}) \text{Log}\left[\left(4+x+x^2\right)\left(43+52x+52x^2 + \sqrt{165}\sqrt{5+4x+4x^2} + 2\sqrt{165}x\sqrt{5+4x+4x^2}\right)\right]}{4\sqrt{165}} \end{aligned}$$

Problem 246: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \frac{3+x}{(1+x^2)\sqrt{1+x+x^2}} dx$$

Optimal (type 3, 56 leaves, 5 steps):

$$-2\sqrt{2} \text{ArcTan}\left[\frac{1-x}{\sqrt{2}\sqrt{1+x+x^2}}\right] + \sqrt{2} \text{ArcTanh}\left[\frac{1+x}{\sqrt{2}\sqrt{1+x+x^2}}\right]$$

Result (type 3, 352 leaves):

$$\begin{aligned} & \left(\frac{1}{4} + \frac{\mathbf{i}}{4}\right) (-1)^{3/4} \left( (4+2\mathbf{i}) \text{ArcTan}\left[\frac{(-7+12\mathbf{i}) + (12+25\mathbf{i})x^3 + 40(-1)^{1/4}\sqrt{1+x+x^2} + x^2 \left((5+28\mathbf{i}) + 20(-1)^{3/4}\sqrt{1+x+x^2}\right)}{((-4+37\mathbf{i}) - (10-30\mathbf{i})\sqrt{2}\sqrt{1+x+x^2})}\right] \right) / \left( (1-36\mathbf{i}) + (32-11\mathbf{i})x + (5+16\mathbf{i})x^2 + (4+25\mathbf{i})x^3 \right) + \\ & (4-2\mathbf{i}) \text{ArcTan}\left[\frac{(-7-12\mathbf{i}) + (12-25\mathbf{i})x^3 + 20(-1)^{1/4}\sqrt{1+x+x^2} + x^2 \left((5-28\mathbf{i}) - 40(-1)^{3/4}\sqrt{1+x+x^2}\right)}{((-4-37\mathbf{i}) + (30+10\mathbf{i})\sqrt{2}\sqrt{1+x+x^2})}\right] / \left( (-49+36\mathbf{i}) - (48-61\mathbf{i})x - (45-64\mathbf{i})x^2 + (4+25\mathbf{i})x^3 \right) + \\ & 2 \text{Log}\left[1+x^2\right] - (1+2\mathbf{i}) \text{Log}\left[\left(5+4\mathbf{i}\right) + \left(8+4\mathbf{i}\right)x + \left(5+4\mathbf{i}\right)x^2 + 8(-1)^{1/4}\sqrt{1+x+x^2} + 4(-1)^{1/4}x\sqrt{1+x+x^2}\right] - \\ & (1-2\mathbf{i}) \text{Log}\left[\left(5+4\mathbf{i}\right) + \left(8+4\mathbf{i}\right)x + \left(5+4\mathbf{i}\right)x^2 + 4(-1)^{1/4}\sqrt{1+x+x^2} + 8(-1)^{1/4}x\sqrt{1+x+x^2}\right] \end{aligned}$$

**Problem 247:** Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \frac{1 + 2x}{\sqrt{-1 + 6x + x^2} (4 + 4x + 3x^2)} dx$$

Optimal (type 3, 70 leaves, 5 steps):

$$-\frac{5 \operatorname{ArcTan}\left[\frac{\sqrt{\frac{7}{2}}(2-x)}{2\sqrt{-1+6x+x^2}}\right]}{6\sqrt{14}} - \frac{\operatorname{ArcTanh}\left[\frac{\sqrt{7}(1+x)}{\sqrt{-1+6x+x^2}}\right]}{3\sqrt{7}}$$

Result (type 3, 991 leaves):

$$\begin{aligned}
 & \frac{1}{8\sqrt{14}} \left( \frac{1}{\sqrt{7+4i\sqrt{2}}} 2(-i+4\sqrt{2}) \operatorname{ArcTan} \left[ \left( 7840 - 5816i\sqrt{2} + 18(112+37i\sqrt{2})x^4 + 3564i\sqrt{7(7+4i\sqrt{2})}\sqrt{-1+6x+x^2} + \right. \right. \right. \\
 & \quad \left. \left. \left. i x^2 \left( 56224i + 29126\sqrt{2} - 99\sqrt{7(7+4i\sqrt{2})}\sqrt{-1+6x+x^2} \right) - 72ix \left( -546i + 265\sqrt{2} - 11\sqrt{7(7+4i\sqrt{2})}\sqrt{-1+6x+x^2} \right) + \right. \right. \right. \\
 & \quad \left. \left. \left. 3x^3 \left( 1456 + 7564i\sqrt{2} - 693i\sqrt{7(7+4i\sqrt{2})}\sqrt{-1+6x+x^2} \right) \right) \right] / \left( 9836i - 5600\sqrt{2} + 36(-1083i + 560\sqrt{2})x + \right. \right. \\
 & \quad \left. \left. \left( -41651i + 78176\sqrt{2} \right)x^2 + \left( -91506i + 61824\sqrt{2} \right)x^3 + 9(-1487i + 896\sqrt{2})x^4 \right) \right] - \frac{1}{\sqrt{-7+4i\sqrt{2}}} \\
 & 2(i+4\sqrt{2}) \operatorname{ArcTanh} \left[ \left( 4(6344i - 700\sqrt{2} + 18(477i + 140\sqrt{2})x + (9847i + 9772\sqrt{2})x^2 + 12(947i + 644\sqrt{2})x^3 + 9(421i + 112\sqrt{2})x^4 \right) \right] / \\
 & \quad \left( -9(112i + 37\sqrt{2})x^4 + 36x \left( 546i + 265\sqrt{2} + 44\sqrt{-98+56i\sqrt{2}}\sqrt{-1+6x+x^2} \right) + \right. \\
 & \quad \left. 3x^3 \left( -728i - 3782\sqrt{2} + 99\sqrt{-98+56i\sqrt{2}}\sqrt{-1+6x+x^2} \right) + 4 \left( -980i + 727\sqrt{2} + 297\sqrt{-98+56i\sqrt{2}}\sqrt{-1+6x+x^2} \right) + \right. \\
 & \quad \left. x^2 \left( 28112i - 14563\sqrt{2} + 1287\sqrt{-98+56i\sqrt{2}}\sqrt{-1+6x+x^2} \right) \right) \right] + \\
 & \frac{(1+4i\sqrt{2}) \operatorname{Log}[9(4+4x+3x^2)^2]}{\sqrt{7+4i\sqrt{2}}} + \frac{(i+4\sqrt{2}) \operatorname{Log}[9(4+4x+3x^2)^2]}{\sqrt{-7+4i\sqrt{2}}} - \frac{1}{\sqrt{-7+4i\sqrt{2}}} \\
 & (i+4\sqrt{2}) \operatorname{Log}[(4+4x+3x^2) \\
 & \quad \left( -101i - 14\sqrt{2} + 2(-2i+7\sqrt{2})x^2 + 9i\sqrt{7(-7+4i\sqrt{2})}\sqrt{-1+6x+x^2} + x \left( 186i + 84\sqrt{2} - 7i\sqrt{7(-7+4i\sqrt{2})}\sqrt{-1+6x+x^2} \right) \right) \right] - \\
 & \frac{1}{\sqrt{7+4i\sqrt{2}}} i(-i+4\sqrt{2}) \operatorname{Log}[(4+4x+3x^2) \left( (-53i+14\sqrt{2})x^2 + 2x \left( -54i+42\sqrt{2} - i\sqrt{98+56i\sqrt{2}}\sqrt{-1+6x+x^2} \right) - \right. \\
 & \quad \left. \left. 2i \left( 26-7i\sqrt{2} + 3\sqrt{98+56i\sqrt{2}}\sqrt{-1+6x+x^2} \right) \right) \right] \right]
 \end{aligned}$$

**Problem 248: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{B + Ax}{(17 - 18x + 5x^2)\sqrt{13 - 22x + 10x^2}} dx$$

Optimal (type 3, 80 leaves, 5 steps):

$$\frac{(2A + B) \operatorname{ArcTan}\left[\frac{\sqrt{35}(2-x)}{\sqrt{13-22x+10x^2}}\right]}{\sqrt{35}} - \frac{(A + B) \operatorname{ArcTanh}\left[\frac{\sqrt{35}(1-x)}{2\sqrt{13-22x+10x^2}}\right]}{2\sqrt{35}}$$

Result (type 3, 1149 leaves):

$$\begin{aligned} & \frac{1}{8\sqrt{35}} \left( ((8 - 2i)A + (4 - 2i)B) \right. \\ & \quad \operatorname{ArcTan}\left[ (4A^2 ((-2494 - 6746i) + (3811 + 15444i)x - (1900 + 11640i)x^2 + (300 + 2800i)x^3) + (2 + 4i)B^2 ((-1843 + 92i) + (3955 + 186i)x - \right. \\ & \quad \left. (2827 + 336i)x^2 + (645 + 110i)x^3) + (4 + 8i)AB ((-3439 - 76i) + (7427 + 942i)x - (5354 + 1092i)x^2 + (1240 + 320i)x^3) \right] / \\ & \quad \left( (1 + 2i)B^2 ((-608 - 1208i) + (395 + 610i)x^3 + (66 - 77i)\sqrt{35}\sqrt{13 - 22x + 10x^2} + \right. \\ & \quad \left. x((1540 + 3036i) - (104 - 103i)\sqrt{35}\sqrt{13 - 22x + 10x^2}) + (4 - 3i)x^2((80 - 549i) + 10\sqrt{35}\sqrt{13 - 22x + 10x^2}) \right) + \\ & \quad A^2 \left( (10987 - 3210i) - (4800 - 2800i)x^3 + (748 + 187i)\sqrt{35}\sqrt{13 - 22x + 10x^2} + \right. \\ & \quad \left. 10x^2((1969 - 892i) + (34 + 17i)\sqrt{35}\sqrt{13 - 22x + 10x^2}) - x((25633 - 9460i) + (1054 + 357i)\sqrt{35}\sqrt{13 - 22x + 10x^2}) \right) + \\ & \quad AB \left( (9519 - 6362i) - (4225 - 4200i)x^3 + (792 + 198i)\sqrt{35}\sqrt{13 - 22x + 10x^2} + \right. \\ & \quad \left. (10 + 5i)x^2 \left( (828 - 1871i) + 36\sqrt{35}\sqrt{13 - 22x + 10x^2} \right) - x \left( (22801 - 16808i) + (1116 + 378i)\sqrt{35}\sqrt{13 - 22x + 10x^2} \right) \right) \left. \right] + \\ & \quad ((-2 + 8i)A - (2 - 4i)B) \operatorname{ArcTanh}\left[ (A^2 ((3594 - 15991i) - (8096 - 43289i)x + (5990 - 39425i)x^2 - (1400 - 12175i)x^3) + \right. \\ & \quad (2 + i)B^2 ((-367 - 3288i) + (1085 + 8506i)x - (1073 + 7336i)x^2 + (355 + 2110i)x^3) + \\ & \quad \left. (4 + 2i)AB ((-1147 - 4952i) + (3185 + 12882i)x - (2993 + 11256i)x^2 + (955 + 3310i)x^3) \right] / \\ & \quad \left( 2 \left( (2 + i)B^2 ((-1208 - 608i) + (610 + 395i)x^3 + (77 - 66i)\sqrt{35}\sqrt{13 - 22x + 10x^2} + \right. \right. \\ & \quad \left. \left. x((3036 + 1540i) - (103 - 104i)\sqrt{35}\sqrt{13 - 22x + 10x^2}) + (3 - 4i)x^2((-80 - 549i) + 10\sqrt{35}\sqrt{13 - 22x + 10x^2}) \right) \right) + \\ & \quad AB \left( (-9519 - 6362i) + (4225 + 4200i)x^3 + (792 - 198i)\sqrt{35}\sqrt{13 - 22x + 10x^2} + \right. \\ & \quad \left. x((22801 + 16808i) - (1116 - 378i)\sqrt{35}\sqrt{13 - 22x + 10x^2}) + (10 - 5i)x^2((-828 - 1871i) + 36\sqrt{35}\sqrt{13 - 22x + 10x^2}) \right) + \\ & \quad A^2 \left( (4800 + 2800i)x^3 + x((25633 + 9460i) - (1054 - 357i)\sqrt{35}\sqrt{13 - 22x + 10x^2}) - \right. \\ & \quad \left. 10ix^2((892 - 1969i) + (17 + 34i)\sqrt{35}\sqrt{13 - 22x + 10x^2}) + (1 - 4i) \left( (109 - 2774i) + (88 + 165i)\sqrt{35}\sqrt{13 - 22x + 10x^2} \right) \right) \left. \right) \left. \right] - \\ & 2A \operatorname{Log}[i(17 - 18x + 5x^2)] - 2B \operatorname{Log}[i(17 - 18x + 5x^2)] + (1 - 4i) \\ & A \\ & \operatorname{Log}[ \end{aligned}$$

$$\begin{aligned}
& (1 + 2i) \left( (-127 - 1566i) + (118 + 2844i)x - (25 + 1350i)x^2 + 68i\sqrt{35}\sqrt{13 - 22x + 10x^2} - 70i\sqrt{35}x\sqrt{13 - 22x + 10x^2} \right) + \\
& (1 - 2i) \text{B Log} \left[ (1 + 2i) \left( (-127 - 1566i) + (118 + 2844i)x - (25 + 1350i)x^2 + 68i\sqrt{35}\sqrt{13 - 22x + 10x^2} - 70i\sqrt{35}x\sqrt{13 - 22x + 10x^2} \right) \right] + \\
& (1 + 4i) \\
& \text{A} \\
& \text{Log} \left[ \right. \\
& \quad (2 + i) \left( (1566 + 127i) - (2844 + 118i)x + (1350 + 25i)x^2 - 68\sqrt{35}\sqrt{13 - 22x + 10x^2} + 70\sqrt{35}x\sqrt{13 - 22x + 10x^2} \right) \left. \right] + \\
& (1 + 2i) \text{B Log} \left[ (2 + i) \left( (1566 + 127i) - (2844 + 118i)x + (1350 + 25i)x^2 - 68\sqrt{35}\sqrt{13 - 22x + 10x^2} + 70\sqrt{35}x\sqrt{13 - 22x + 10x^2} \right) \right] \left. \right]
\end{aligned}$$

**Problem 249: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{-2 + x}{(17 - 18x + 5x^2)\sqrt{13 - 22x + 10x^2}} dx$$

Optimal (type 3, 38 leaves, 2 steps):

$$\frac{\text{ArcTanh} \left[ \frac{\sqrt{35}(1-x)}{2\sqrt{13-22x+10x^2}} \right]}{2\sqrt{35}}$$

Result (type 3, 410 leaves):

$$\begin{aligned}
& \frac{1}{8\sqrt{35}} \left( -2i \text{ArcTan} \left[ (4((-2 + 2i) + 5x)(13 - 22x + 10x^2)) \right] / \left( (-819 - 182i) + 350x^3 + (44 + 11i)\sqrt{35}\sqrt{13 - 22x + 10x^2} + \right. \right. \\
& \quad \left. \left. x \left( (1841 + 308i) - (62 + 21i)\sqrt{35}\sqrt{13 - 22x + 10x^2} \right) + 10x^2 \left( (-140 - 14i) + (2 + i)\sqrt{35}\sqrt{13 - 22x + 10x^2} \right) \right) \right] - \\
& 2i \text{ArcTan} \left[ ((7 + 14i)((-169 - 116i) + (419 + 218i)x - (335 + 140i)x^2 + (85 + 30i)x^3)) \right] / \\
& \quad \left( (-1638 + 364i) + 700x^3 - (88 - 22i)\sqrt{35}\sqrt{13 - 22x + 10x^2} + 20ix^2 \left( (14 + 140i) + (1 + 2i)\sqrt{35}\sqrt{13 - 22x + 10x^2} \right) + \right. \\
& \quad \left. (4 - 2i)x \left( (798 + 245i) + (29 + 4i)\sqrt{35}\sqrt{13 - 22x + 10x^2} \right) \right) \left. \right] + 2 \text{Log} [i(17 - 18x + 5x^2)] - \\
& \text{Log} \left[ (1 + 2i) \left( (1566 - 127i) - (2844 - 118i)x + (1350 - 25i)x^2 - 68\sqrt{35}\sqrt{13 - 22x + 10x^2} + 70\sqrt{35}x\sqrt{13 - 22x + 10x^2} \right) \right] - \\
& \text{Log} \left[ (2 + i) \left( (1566 + 127i) - (2844 + 118i)x + (1350 + 25i)x^2 - 68\sqrt{35}\sqrt{13 - 22x + 10x^2} + 70\sqrt{35}x\sqrt{13 - 22x + 10x^2} \right) \right]
\end{aligned}$$

### Problem 260: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \frac{x^2 (2 - \sqrt{1+x^2})}{\sqrt{1+x^2} (1-x^3 + (1+x^2)^{3/2})} dx$$

Optimal (type 3, 136 leaves, 32 steps):

$$\frac{8x}{9} - \frac{x^2}{6} + \frac{8\sqrt{1+x^2}}{9} - \frac{1}{6}x\sqrt{1+x^2} - \frac{41\text{ArcSinh}[x]}{54} + \frac{4}{27}\sqrt{2}\text{ArcTan}\left[\frac{1+3x}{2\sqrt{2}}\right] + \frac{4}{27}\sqrt{2}\text{ArcTan}\left[\frac{1+x}{\sqrt{2}\sqrt{1+x^2}}\right] + \frac{7}{27}\text{ArcTanh}\left[\frac{1-x}{2\sqrt{1+x^2}}\right] - \frac{7}{54}\text{Log}[3+2x+3x^2]$$

Result (type 3, 947 leaves):

$$\frac{1}{108} \left( 96x - 18x^2 - 6(-16+3x)\sqrt{1+x^2} - 82\text{ArcSinh}[x] + 16\sqrt{2}\text{ArcTan}\left[\frac{1+3x}{2\sqrt{2}}\right] - \frac{1}{\sqrt{1+2i\sqrt{2}}} 2i(-i+11\sqrt{2}) \right. \\ \left. \text{ArcTan}\left[ \left( 2(169(7-4i\sqrt{2}) - 1716i(-i+2\sqrt{2})x + (-4622-5032i\sqrt{2})x^2 - 1716i(-i+2\sqrt{2})x^3 + (-1449-4356i\sqrt{2})x^4 \right) \right] / \right. \\ \left. \left( -559(-8i+7\sqrt{2}) + 9(-88i+383\sqrt{2})x^4 + 12x(230(4i+\sqrt{2}) + 729\sqrt{1+2i\sqrt{2}}\sqrt{1+x^2}) + \right. \right. \\ \left. \left. 12x^3(230(4i+\sqrt{2}) + 729\sqrt{1+2i\sqrt{2}}\sqrt{1+x^2}) + x^2(3680i-862\sqrt{2} + 5832\sqrt{1+2i\sqrt{2}}\sqrt{1+x^2}) \right) \right] + \frac{1}{\sqrt{-1+2i\sqrt{2}}} 2(i+11\sqrt{2}) \\ \text{ArcTan}\left[ \left( 559(8-7i\sqrt{2}) + 9i(88i+383\sqrt{2})x^4 + 6561i\sqrt{-2+4i\sqrt{2}}\sqrt{1+x^2} + 3x^3(920(4+i\sqrt{2}) - 729i\sqrt{-2+4i\sqrt{2}}\sqrt{1+x^2}) + \right. \right. \\ \left. \left. 3x(920(4+i\sqrt{2}) + 729i\sqrt{-2+4i\sqrt{2}}\sqrt{1+x^2}) + x^2(3680-862i\sqrt{2} + 5103i\sqrt{-2+4i\sqrt{2}}\sqrt{1+x^2}) \right) \right] / \\ \left. \left( 17317i + 1352\sqrt{2} + 3432(i+2\sqrt{2})x + 2(19931i+5032\sqrt{2})x^2 + 3432(i+2\sqrt{2})x^3 + 9(2509i+968\sqrt{2})x^4 \right) \right] - \\ 14\text{Log}[3+2x+3x^2] - \frac{(-i+11\sqrt{2})\text{Log}[9(3+2x+3x^2)^2]}{\sqrt{1+2i\sqrt{2}}} + \frac{i(i+11\sqrt{2})\text{Log}[9(3+2x+3x^2)^2]}{\sqrt{-1+2i\sqrt{2}}} + \frac{1}{\sqrt{-1+2i\sqrt{2}}} \\ \left. \left( 1-11i\sqrt{2} \right) \text{Log}\left[ (3+2x+3x^2) \left( -7i+4\sqrt{2} + (-7i+4\sqrt{2})x^2 - 2ix(-3+4\sqrt{-1+2i\sqrt{2}}\sqrt{1+x^2}) \right) \right] \right] + \frac{1}{\sqrt{1+2i\sqrt{2}}} \\ \left. \left( -i+11\sqrt{2} \right) \text{Log}\left[ (3+2x+3x^2) \left( -11i+4\sqrt{2} + (-11i+4\sqrt{2})x^2 - 6i\sqrt{2+4i\sqrt{2}}\sqrt{1+x^2} + 2ix(3+\sqrt{2+4i\sqrt{2}}\sqrt{1+x^2}) \right) \right] \right] \right)$$



### Problem 264: Result more than twice size of optimal antiderivative.

$$\int \frac{1}{(-1+x^2)\sqrt{2x+x^2}} dx$$

Optimal (type 3, 49 leaves, 5 steps):

$$-\frac{1}{2} \operatorname{ArcTan}[\sqrt{2x+x^2}] - \frac{\operatorname{ArcTanh}\left[\frac{1+2x}{\sqrt{3}\sqrt{2x+x^2}}\right]}{2\sqrt{3}}$$

Result (type 3, 113 leaves):

$$\frac{1}{6\sqrt{x(2+x)}} \sqrt{x}\sqrt{2+x} \left( -6 \operatorname{ArcTan}\left[\sqrt{\frac{x}{2+x}}\right] + \sqrt{3} \left( \operatorname{Log}[1-\sqrt{x}] - \operatorname{Log}[1+\sqrt{x}] + \operatorname{Log}[2-\sqrt{x}+\sqrt{3}\sqrt{2+x}] - \operatorname{Log}[2+\sqrt{x}+\sqrt{3}\sqrt{2+x}] \right) \right)$$

### Problem 294: Result unnecessarily involves higher level functions.

$$\int \frac{(-1+3x)^{4/3}}{x^2} dx$$

Optimal (type 3, 71 leaves, 6 steps):

$$12(-1+3x)^{1/3} - \frac{(-1+3x)^{4/3}}{x} + 4\sqrt{3} \operatorname{ArcTan}\left[\frac{1-2(-1+3x)^{1/3}}{\sqrt{3}}\right] + 2 \operatorname{Log}[x] - 6 \operatorname{Log}\left[1+(-1+3x)^{1/3}\right]$$

Result (type 5, 59 leaves):

$$\frac{-1-6x+27x^2+2 \times 3^{1/3} \left(3-\frac{1}{x}\right)^{2/3} x \operatorname{Hypergeometric2F1}\left[\frac{2}{3}, \frac{2}{3}, \frac{5}{3}, \frac{1}{3x}\right]}{x(-1+3x)^{2/3}}$$

### Problem 296: Result unnecessarily involves higher level functions.

$$\int \frac{(1-2x^{1/3})^{3/4}}{x} dx$$

Optimal (type 3, 48 leaves, 6 steps):

$$4(1-2x^{1/3})^{3/4} + 6 \operatorname{ArcTan}\left[(1-2x^{1/3})^{1/4}\right] - 6 \operatorname{ArcTanh}\left[(1-2x^{1/3})^{1/4}\right]$$

Result (type 5, 62 leaves):

$$\frac{4 - 8 x^{1/3} - 6 \times 2^{3/4} \left(2 - \frac{1}{x^{1/3}}\right)^{1/4} \text{Hypergeometric2F1}\left[\frac{1}{4}, \frac{1}{4}, \frac{5}{4}, \frac{1}{2 x^{1/3}}\right]}{(1 - 2 x^{1/3})^{1/4}}$$

**Problem 298: Result unnecessarily involves higher level functions.**

$$\int \frac{(-1 + 2 \sqrt{x})^{5/4}}{x^2} dx$$

Optimal (type 3, 193 leaves, 13 steps):

$$\begin{aligned} & -\frac{(-1 + 2 \sqrt{x})^{5/4}}{x} - \frac{5 (-1 + 2 \sqrt{x})^{1/4}}{2 \sqrt{x}} - \frac{5 \text{ArcTan}\left[1 - \sqrt{2} (-1 + 2 \sqrt{x})^{1/4}\right]}{2 \sqrt{2}} + \frac{5 \text{ArcTan}\left[1 + \sqrt{2} (-1 + 2 \sqrt{x})^{1/4}\right]}{2 \sqrt{2}} \\ & - \frac{5 \text{Log}\left[1 - \sqrt{2} (-1 + 2 \sqrt{x})^{1/4} + \sqrt{-1 + 2 \sqrt{x}}\right]}{4 \sqrt{2}} + \frac{5 \text{Log}\left[1 + \sqrt{2} (-1 + 2 \sqrt{x})^{1/4} + \sqrt{-1 + 2 \sqrt{x}}\right]}{4 \sqrt{2}} \end{aligned}$$

Result (type 5, 72 leaves):

$$\frac{-6 + 39 \sqrt{x} - 54 x - 5 \times 2^{1/4} \left(2 - \frac{1}{\sqrt{x}}\right)^{3/4} x \text{Hypergeometric2F1}\left[\frac{3}{4}, \frac{3}{4}, \frac{7}{4}, \frac{1}{2 \sqrt{x}}\right]}{6 (-1 + 2 \sqrt{x})^{3/4} x}$$

**Problem 301: Result unnecessarily involves higher level functions.**

$$\int \frac{1}{x (-27 + 2 x^7)^{2/3}} dx$$

Optimal (type 3, 59 leaves, 5 steps):

$$-\frac{\text{ArcTan}\left[\frac{3-2(-27+2x^7)^{1/3}}{3\sqrt{3}}\right]}{21\sqrt{3}} - \frac{\text{Log}[x]}{18} + \frac{1}{42} \text{Log}\left[3 + (-27 + 2 x^7)^{1/3}\right]$$

Result (type 5, 43 leaves):

$$-\frac{3 \left(2 - \frac{27}{x^7}\right)^{2/3} \text{Hypergeometric2F1}\left[\frac{2}{3}, \frac{2}{3}, \frac{5}{3}, \frac{27}{2 x^7}\right]}{14 (-54 + 4 x^7)^{2/3}}$$

### Problem 302: Result unnecessarily involves higher level functions.

$$\int \frac{(1+x^7)^{2/3}}{x^8} dx$$

Optimal (type 3, 70 leaves, 6 steps):

$$-\frac{(1+x^7)^{2/3}}{7x^7} + \frac{2 \operatorname{ArcTan}\left[\frac{1+2(1+x^7)^{1/3}}{\sqrt{3}}\right]}{7\sqrt{3}} - \frac{\operatorname{Log}[x]}{3} + \frac{1}{7} \operatorname{Log}\left[1 - (1+x^7)^{1/3}\right]$$

Result (type 5, 54 leaves):

$$-\frac{(1+x^7)^{2/3}}{7x^7} - \frac{2\left(1+\frac{1}{x^7}\right)^{1/3} \operatorname{Hypergeometric2F1}\left[\frac{1}{3}, \frac{1}{3}, \frac{4}{3}, -\frac{1}{x^7}\right]}{7(1+x^7)^{1/3}}$$

### Problem 303: Result unnecessarily involves higher level functions.

$$\int \frac{(3+4x^4)^{1/4}}{x^2} dx$$

Optimal (type 3, 68 leaves, 5 steps):

$$-\frac{(3+4x^4)^{1/4}}{x} - \frac{\operatorname{ArcTan}\left[\frac{\sqrt{2}x}{(3+4x^4)^{1/4}}\right]}{\sqrt{2}} + \frac{\operatorname{ArcTanh}\left[\frac{\sqrt{2}x}{(3+4x^4)^{1/4}}\right]}{\sqrt{2}}$$

Result (type 5, 46 leaves):

$$-\frac{(3+4x^4)^{1/4}}{x} + \frac{4x^3 \operatorname{Hypergeometric2F1}\left[\frac{3}{4}, \frac{3}{4}, \frac{7}{4}, -\frac{4x^4}{3}\right]}{3 \times 3^{3/4}}$$

### Problem 304: Result unnecessarily involves higher level functions.

$$\int x^2 (3+4x^4)^{5/4} dx$$

Optimal (type 3, 93 leaves, 6 steps):

$$\frac{15}{32} x^3 (3+4x^4)^{1/4} + \frac{1}{8} x^3 (3+4x^4)^{5/4} - \frac{45 \operatorname{ArcTan}\left[\frac{\sqrt{2}x}{(3+4x^4)^{1/4}}\right]}{128\sqrt{2}} + \frac{45 \operatorname{ArcTanh}\left[\frac{\sqrt{2}x}{(3+4x^4)^{1/4}}\right]}{128\sqrt{2}}$$

Result (type 5, 51 leaves):

$$\frac{1}{32} x^3 \left( (3 + 4 x^4)^{1/4} (27 + 16 x^4) + 5 \times 3^{1/4} \text{Hypergeometric2F1} \left[ \frac{3}{4}, \frac{3}{4}, \frac{7}{4}, -\frac{4 x^4}{3} \right] \right)$$

**Problem 305: Result unnecessarily involves higher level functions.**

$$\int x^6 (3 + 4 x^4)^{1/4} dx$$

Optimal (type 3, 93 leaves, 6 steps):

$$\frac{3}{128} x^3 (3 + 4 x^4)^{1/4} + \frac{1}{8} x^7 (3 + 4 x^4)^{1/4} + \frac{27 \text{ArcTan} \left[ \frac{\sqrt{2} x}{(3 + 4 x^4)^{1/4}} \right]}{512 \sqrt{2}} - \frac{27 \text{ArcTanh} \left[ \frac{\sqrt{2} x}{(3 + 4 x^4)^{1/4}} \right]}{512 \sqrt{2}}$$

Result (type 5, 51 leaves):

$$\frac{1}{128} x^3 \left( (3 + 4 x^4)^{1/4} (3 + 16 x^4) - 3 \times 3^{1/4} \text{Hypergeometric2F1} \left[ \frac{3}{4}, \frac{3}{4}, \frac{7}{4}, -\frac{4 x^4}{3} \right] \right)$$

**Problem 306: Result unnecessarily involves higher level functions.**

$$\int (x (1 - x^2))^{1/3} dx$$

Optimal (type 3, 93 leaves, ? steps):

$$\frac{1}{2} x (x (1 - x^2))^{1/3} + \frac{\text{ArcTan} \left[ \frac{2 x - (x (1 - x^2))^{1/3}}{\sqrt{3} (x (1 - x^2))^{1/3}} \right]}{2 \sqrt{3}} + \frac{\text{Log}[x]}{12} - \frac{1}{4} \text{Log} \left[ x + (x (1 - x^2))^{1/3} \right]$$

Result (type 5, 56 leaves):

$$\frac{x (x - x^3)^{1/3} \left( -2 + 2 x^2 - (1 - x^2)^{2/3} \text{Hypergeometric2F1} \left[ \frac{2}{3}, \frac{2}{3}, \frac{5}{3}, x^2 \right] \right)}{4 (-1 + x^2)}$$

**Problem 311: Result more than twice size of optimal antiderivative.**

$$\int \frac{-1 + x^2}{x \sqrt{1 + 3 x^2 + x^4}} dx$$

Optimal (type 3, 21 leaves, 3 steps):

$$\text{ArcTanh} \left[ \frac{1 + x^2}{\sqrt{1 + 3 x^2 + x^4}} \right]$$

Result (type 3, 59 leaves):

$$\frac{1}{2} \left( -\text{Log}[x^2] + \text{Log}\left[3 + 2x^2 + 2\sqrt{1 + 3x^2 + x^4}\right] + \text{Log}\left[2 + 3x^2 + 2\sqrt{1 + 3x^2 + x^4}\right] \right)$$

Problem 319: Unable to integrate problem.

$$\int \frac{1}{(3x + 3x^2 + x^3)(3 + 3x + 3x^2 + x^3)^{1/3}} dx$$

Optimal (type 3, 123 leaves, 9 steps):

$$-\frac{\text{ArcTan}\left[\frac{1}{\sqrt{3}} + \frac{2(1+x)}{3^{1/6}(2+(1+x)^3)^{1/3}}\right]}{3^{5/6}} + \frac{\text{Log}\left[1 - \frac{3^{1/3}(1+x)}{(2+(1+x)^3)^{1/3}}\right]}{3 \times 3^{1/3}} - \frac{\text{Log}\left[1 + \frac{3^{2/3}(1+x)^2}{(2+(1+x)^3)^{2/3}} + \frac{3^{1/3}(1+x)}{(2+(1+x)^3)^{1/3}}\right]}{6 \times 3^{1/3}}$$

Result (type 8, 34 leaves):

$$\int \frac{1}{(3x + 3x^2 + x^3)(3 + 3x + 3x^2 + x^3)^{1/3}} dx$$

Problem 320: Result unnecessarily involves higher level functions.

$$\int \frac{1-x^2}{(1+x^2)\sqrt{1+x^4}} dx$$

Optimal (type 3, 23 leaves, 2 steps):

$$\frac{\text{ArcTan}\left[\frac{\sqrt{2}x}{\sqrt{1+x^4}}\right]}{\sqrt{2}}$$

Result (type 4, 40 leaves):

$$(-1)^{1/4} \left( \text{EllipticF}\left[i \text{ArcSinh}\left[(-1)^{1/4}x\right], -1\right] - 2 \text{EllipticPi}\left[-i, i \text{ArcSinh}\left[(-1)^{1/4}x\right], -1\right] \right)$$

Problem 321: Result unnecessarily involves higher level functions.

$$\int \frac{1+x^2}{(1-x^2)\sqrt{1+x^4}} dx$$

Optimal (type 3, 23 leaves, 2 steps):

$$\frac{\text{ArcTanh}\left[\frac{\sqrt{2}x}{\sqrt{1+x^4}}\right]}{\sqrt{2}}$$

Result (type 4, 36 leaves):

$$(-1)^{1/4} \left( \text{EllipticF}\left[i \text{ArcSinh}\left[(-1)^{1/4}x\right], -1\right] - 2 \text{EllipticPi}\left[i, \text{ArcSin}\left[(-1)^{3/4}x\right], -1\right] \right)$$

**Problem 324: Unable to integrate problem.**

$$\int \frac{1+x^2}{(1-x^2)\sqrt{1+x^2+x^4}} dx$$

Optimal (type 3, 26 leaves, 2 steps):

$$\frac{\text{ArcTanh}\left[\frac{\sqrt{3}x}{\sqrt{1+x^2+x^4}}\right]}{\sqrt{3}}$$

Result (type 8, 29 leaves):

$$\int \frac{1+x^2}{(1-x^2)\sqrt{1+x^2+x^4}} dx$$

**Problem 325: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{1-x^2}{(1+x^2)\sqrt{1+x^2+x^4}} dx$$

Optimal (type 3, 15 leaves, 2 steps):

$$\text{ArcTan}\left[\frac{x}{\sqrt{1+x^2+x^4}}\right]$$

Result (type 4, 94 leaves):

$$-\frac{1}{\sqrt{1+x^2+x^4}} (-1)^{2/3} \sqrt{1+(-1)^{1/3}x^2} \sqrt{1-(-1)^{2/3}x^2} \left( \text{EllipticF}\left[i \text{ArcSinh}\left[(-1)^{5/6}x\right], (-1)^{2/3}\right] + 2 \text{EllipticPi}\left[(-1)^{1/3}, -i \text{ArcSinh}\left[(-1)^{5/6}x\right], (-1)^{2/3}\right] \right)$$

**Problem 327: Result unnecessarily involves higher level functions and more than twice size of optimal**

antiderivative.

$$\int \frac{1-x^2}{(1+2ax+x^2)\sqrt{1+2ax+2bx^2+2ax^3+x^4}} dx$$

Optimal (type 3, 74 leaves, 1 step):

$$\frac{\text{ArcTan}\left[\frac{a+2(1+a^2-b)x+ax^2}{\sqrt{2}\sqrt{1-b}\sqrt{1+2ax+2bx^2+2ax^3+x^4}}\right]}{\sqrt{2}\sqrt{1-b}}$$

Result (type 4, 17955 leaves):

$$\begin{aligned} & \left(2a(x - \text{Root}[1+2a\#1+2b\#1^2+2a\#1^3+\#1^4, 2])\right)^2 \\ & \left(\text{EllipticF}\left[\text{ArcSin}\left[\sqrt{\left(\left(x - \text{Root}[1+2a\#1+2b\#1^2+2a\#1^3+\#1^4, 1]\right)\left(\text{Root}[1+2a\#1+2b\#1^2+2a\#1^3+\#1^4, 2] - \right.\right.\right.\right. \right. \\ & \quad \left.\left.\left.\left.\text{Root}[1+2a\#1+2b\#1^2+2a\#1^3+\#1^4, 4]\right)\right)\right) / \left(\left(x - \text{Root}[1+2a\#1+2b\#1^2+2a\#1^3+\#1^4, 2]\right)\right.\right. \right. \\ & \quad \left.\left.\left.\left.\left(\text{Root}[1+2a\#1+2b\#1^2+2a\#1^3+\#1^4, 1] - \text{Root}[1+2a\#1+2b\#1^2+2a\#1^3+\#1^4, 4]\right)\right)\right)\right], \right. \\ & \quad - \left(\left(\left(\text{Root}[1+2a\#1+2b\#1^2+2a\#1^3+\#1^4, 2] - \text{Root}[1+2a\#1+2b\#1^2+2a\#1^3+\#1^4, 3]\right)\right.\right. \\ & \quad \left.\left.\left(\text{Root}[1+2a\#1+2b\#1^2+2a\#1^3+\#1^4, 1] - \text{Root}[1+2a\#1+2b\#1^2+2a\#1^3+\#1^4, 4]\right)\right)\right) / \\ & \quad \left(\left(-\text{Root}[1+2a\#1+2b\#1^2+2a\#1^3+\#1^4, 1] + \text{Root}[1+2a\#1+2b\#1^2+2a\#1^3+\#1^4, 3]\right)\left(\text{Root}[1+2a\#1+2b\#1^2+2a\#1^3+\#1^4, \right.\right. \\ & \quad \left.\left.2] - \text{Root}[1+2a\#1+2b\#1^2+2a\#1^3+\#1^4, 4]\right)\right)\right) \left(-a + \sqrt{-1+a^2} - \text{Root}[1+2a\#1+2b\#1^2+2a\#1^3+\#1^4, 1]\right) - \\ & \text{EllipticPi}\left[\left(\left(a - \sqrt{-1+a^2} + \text{Root}[1+2a\#1+2b\#1^2+2a\#1^3+\#1^4, 2]\right)\left(-\text{Root}[1+2a\#1+2b\#1^2+2a\#1^3+\#1^4, 1] + \right.\right. \right. \\ & \quad \left.\left.\text{Root}[1+2a\#1+2b\#1^2+2a\#1^3+\#1^4, 4]\right)\right) / \left(\left(a - \sqrt{-1+a^2} + \text{Root}[1+2a\#1+2b\#1^2+2a\#1^3+\#1^4, 1]\right)\right) \\ & \quad \left(-\text{Root}[1+2a\#1+2b\#1^2+2a\#1^3+\#1^4, 2] + \text{Root}[1+2a\#1+2b\#1^2+2a\#1^3+\#1^4, 4]\right)\right], \\ & \text{ArcSin}\left[\sqrt{\left(\left(x - \text{Root}[1+2a\#1+2b\#1^2+2a\#1^3+\#1^4, 1]\right)\left(\text{Root}[1+2a\#1+2b\#1^2+2a\#1^3+\#1^4, 2] - \right.\right.\right. \right. \\ & \quad \left.\left.\left.\left.\text{Root}[1+2a\#1+2b\#1^2+2a\#1^3+\#1^4, 4]\right)\right)\right) / \left(\left(x - \text{Root}[1+2a\#1+2b\#1^2+2a\#1^3+\#1^4, 2]\right)\right.\right. \right. \\ & \quad \left.\left.\left.\left.\left(\text{Root}[1+2a\#1+2b\#1^2+2a\#1^3+\#1^4, 1] - \text{Root}[1+2a\#1+2b\#1^2+2a\#1^3+\#1^4, 4]\right)\right)\right)\right], \right. \\ & \quad - \left(\left(\left(\text{Root}[1+2a\#1+2b\#1^2+2a\#1^3+\#1^4, 2] - \text{Root}[1+2a\#1+2b\#1^2+2a\#1^3+\#1^4, 3]\right)\right.\right. \\ & \quad \left.\left.\left(\text{Root}[1+2a\#1+2b\#1^2+2a\#1^3+\#1^4, 1] - \text{Root}[1+2a\#1+2b\#1^2+2a\#1^3+\#1^4, 4]\right)\right)\right) / \\ & \quad \left(\left(-\text{Root}[1+2a\#1+2b\#1^2+2a\#1^3+\#1^4, 1] + \text{Root}[1+2a\#1+2b\#1^2+2a\#1^3+\#1^4, 3]\right)\right. \\ & \quad \left.\left(\text{Root}[1+2a\#1+2b\#1^2+2a\#1^3+\#1^4, 2] - \text{Root}[1+2a\#1+2b\#1^2+2a\#1^3+\#1^4, 4]\right)\right)\right) \\ & \quad \left(-\text{Root}[1+2a\#1+2b\#1^2+2a\#1^3+\#1^4, 1] + \text{Root}[1+2a\#1+2b\#1^2+2a\#1^3+\#1^4, 2]\right) \\ & \quad \sqrt{\left(\left(-\text{Root}[1+2a\#1+2b\#1^2+2a\#1^3+\#1^4, 1] + \text{Root}[1+2a\#1+2b\#1^2+2a\#1^3+\#1^4, 2]\right)\right. \\ & \quad \left.\left(x - \text{Root}[1+2a\#1+2b\#1^2+2a\#1^3+\#1^4, 3]\right)\right) / \left(\left(x - \text{Root}[1+2a\#1+2b\#1^2+2a\#1^3+\#1^4, 2]\right)\right. \\ & \quad \left.\left(-\text{Root}[1+2a\#1+2b\#1^2+2a\#1^3+\#1^4, 1] + \text{Root}[1+2a\#1+2b\#1^2+2a\#1^3+\#1^4, 3]\right)\right) \\ & \quad \sqrt{\left(\left(x - \text{Root}[1+2a\#1+2b\#1^2+2a\#1^3+\#1^4, 1]\right)\left(\text{Root}[1+2a\#1+2b\#1^2+2a\#1^3+\#1^4, 2] - \right.\right. \end{aligned}$$























$$\text{Root}\left[1 + 2 a \#1 + 2 b \#1^2 + 2 a \#1^3 + \#1^4 \&, 4\right]$$

**Problem 338: Result more than twice size of optimal antiderivative.**

$$\int \csc [x]^7 dx$$

Optimal (type 3, 36 leaves, 4 steps):

$$-\frac{5}{16} \text{ArcTanh}[\cos [x]] - \frac{5}{16} \cot [x] \csc [x] - \frac{5}{24} \cot [x] \csc [x]^3 - \frac{1}{6} \cot [x] \csc [x]^5$$

Result (type 3, 95 leaves):

$$-\frac{5}{64} \csc \left[\frac{x}{2}\right]^2 - \frac{1}{64} \csc \left[\frac{x}{2}\right]^4 - \frac{1}{384} \csc \left[\frac{x}{2}\right]^6 - \frac{5}{16} \log \left[\cos \left[\frac{x}{2}\right]\right] + \frac{5}{16} \log \left[\sin \left[\frac{x}{2}\right]\right] + \frac{5}{64} \sec \left[\frac{x}{2}\right]^2 + \frac{1}{64} \sec \left[\frac{x}{2}\right]^4 + \frac{1}{384} \sec \left[\frac{x}{2}\right]^6$$

**Problem 355: Result more than twice size of optimal antiderivative.**

$$\int \cot [x]^3 \csc [x] dx$$

Optimal (type 3, 11 leaves, 2 steps):

$$\csc [x] - \frac{\csc [x]^3}{3}$$

Result (type 3, 57 leaves):

$$\frac{5}{12} \cot \left[\frac{x}{2}\right] - \frac{1}{24} \cot \left[\frac{x}{2}\right] \csc \left[\frac{x}{2}\right]^2 + \frac{5}{12} \tan \left[\frac{x}{2}\right] - \frac{1}{24} \sec \left[\frac{x}{2}\right]^2 \tan \left[\frac{x}{2}\right]$$

**Problem 357: Result more than twice size of optimal antiderivative.**

$$\int \cot [x]^2 \csc [x]^3 dx$$

Optimal (type 3, 26 leaves, 3 steps):

$$\frac{1}{8} \text{ArcTanh}[\cos [x]] + \frac{1}{8} \cot [x] \csc [x] - \frac{1}{4} \cot [x] \csc [x]^3$$

Result (type 3, 71 leaves):

$$\frac{1}{32} \csc \left[\frac{x}{2}\right]^2 - \frac{1}{64} \csc \left[\frac{x}{2}\right]^4 + \frac{1}{8} \log \left[\cos \left[\frac{x}{2}\right]\right] - \frac{1}{8} \log \left[\sin \left[\frac{x}{2}\right]\right] - \frac{1}{32} \sec \left[\frac{x}{2}\right]^2 + \frac{1}{64} \sec \left[\frac{x}{2}\right]^4$$

**Problem 361: Result more than twice size of optimal antiderivative.**

$$\int \cot [x]^4 \operatorname{Csc}[x]^3 dx$$

Optimal (type 3, 38 leaves, 4 steps):

$$-\frac{1}{16} \operatorname{ArcTanh}[\cos [x]] - \frac{1}{16} \cot [x] \operatorname{Csc}[x] + \frac{1}{8} \cot [x] \operatorname{Csc}[x]^3 - \frac{1}{6} \cot [x]^3 \operatorname{Csc}[x]^3$$

Result (type 3, 95 leaves):

$$-\frac{1}{64} \operatorname{Csc}\left[\frac{x}{2}\right]^2 + \frac{1}{64} \operatorname{Csc}\left[\frac{x}{2}\right]^4 - \frac{1}{384} \operatorname{Csc}\left[\frac{x}{2}\right]^6 - \frac{1}{16} \operatorname{Log}\left[\cos\left[\frac{x}{2}\right]\right] + \frac{1}{16} \operatorname{Log}\left[\sin\left[\frac{x}{2}\right]\right] + \frac{1}{64} \operatorname{Sec}\left[\frac{x}{2}\right]^2 - \frac{1}{64} \operatorname{Sec}\left[\frac{x}{2}\right]^4 + \frac{1}{384} \operatorname{Sec}\left[\frac{x}{2}\right]^6$$

**Problem 367: Result more than twice size of optimal antiderivative.**

$$\int \cos [4 x] \operatorname{Sec}[x] dx$$

Optimal (type 3, 12 leaves, 4 steps):

$$\operatorname{ArcTanh}[\sin [x]] - \frac{8 \sin [x]^3}{3}$$

Result (type 3, 45 leaves):

$$-\operatorname{Log}\left[\cos\left[\frac{x}{2}\right] - \sin\left[\frac{x}{2}\right]\right] + \operatorname{Log}\left[\cos\left[\frac{x}{2}\right] + \sin\left[\frac{x}{2}\right]\right] - 2 \sin [x] + \frac{2}{3} \sin [3 x]$$

**Problem 369: Result more than twice size of optimal antiderivative.**

$$\int \cos [4 x] \operatorname{Sec}[x]^5 dx$$

Optimal (type 3, 26 leaves, 4 steps):

$$\frac{35}{8} \operatorname{ArcTanh}[\sin [x]] - \frac{29}{8} \operatorname{Sec}[x] \tan [x] + \frac{1}{4} \operatorname{Sec}[x]^3 \tan [x]$$

Result (type 3, 58 leaves):

$$\frac{1}{16} \left( -70 \operatorname{Log}\left[\cos\left[\frac{x}{2}\right] - \sin\left[\frac{x}{2}\right]\right] + 70 \operatorname{Log}\left[\cos\left[\frac{x}{2}\right] + \sin\left[\frac{x}{2}\right]\right] - \frac{1}{2} \operatorname{Sec}[x]^4 (21 \sin [x] + 29 \sin [3 x]) \right)$$

### Problem 383: Result more than twice size of optimal antiderivative.

$$\int \cos [x]^2 \operatorname{Sec} [3 x] \, dx$$

Optimal (type 3, 9 leaves, 2 steps):

$$\frac{1}{2} \operatorname{ArcTanh} [2 \sin [x]]$$

Result (type 3, 23 leaves):

$$-\frac{1}{4} \operatorname{Log} [1 - 2 \sin [x]] + \frac{1}{4} \operatorname{Log} [1 + 2 \sin [x]]$$

### Problem 384: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \operatorname{Sec} [2 x] \sin [x] \, dx$$

Optimal (type 3, 15 leaves, 2 steps):

$$\frac{\operatorname{ArcTanh} [\sqrt{2} \cos [x]]}{\sqrt{2}}$$

Result (type 3, 174 leaves):

$$\frac{1}{4\sqrt{2}} \left( 2i \operatorname{ArcTan} \left[ \frac{\cos \left[ \frac{x}{2} \right] - (-1 + \sqrt{2}) \sin \left[ \frac{x}{2} \right]}{(1 + \sqrt{2}) \cos \left[ \frac{x}{2} \right] - \sin \left[ \frac{x}{2} \right]} \right] - 2i \operatorname{ArcTan} \left[ \frac{\cos \left[ \frac{x}{2} \right] - (1 + \sqrt{2}) \sin \left[ \frac{x}{2} \right]}{(-1 + \sqrt{2}) \cos \left[ \frac{x}{2} \right] - \sin \left[ \frac{x}{2} \right]} \right] + \right. \\ \left. 4 \operatorname{ArcTanh} [\sqrt{2} + \tan \left[ \frac{x}{2} \right]] - \operatorname{Log} [2 - \sqrt{2} \cos [x] - \sqrt{2} \sin [x]] + \operatorname{Log} [2 + \sqrt{2} \cos [x] - \sqrt{2} \sin [x]] \right)$$

### Problem 388: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \operatorname{Csc} [4 x] \sin [x] \, dx$$

Optimal (type 3, 26 leaves, 4 steps):

$$-\frac{1}{4} \operatorname{ArcTanh} [\sin [x]] + \frac{\operatorname{ArcTanh} [\sqrt{2} \sin [x]]}{2\sqrt{2}}$$

Result (type 3, 218 leaves):

$$\frac{1}{8\sqrt{2}} \left( -2i \operatorname{ArcTan} \left[ \frac{\cos\left[\frac{x}{2}\right] - (-1 + \sqrt{2}) \sin\left[\frac{x}{2}\right]}{(1 + \sqrt{2}) \cos\left[\frac{x}{2}\right] - \sin\left[\frac{x}{2}\right]} \right] - 2i \operatorname{ArcTan} \left[ \frac{\cos\left[\frac{x}{2}\right] - (1 + \sqrt{2}) \sin\left[\frac{x}{2}\right]}{(-1 + \sqrt{2}) \cos\left[\frac{x}{2}\right] - \sin\left[\frac{x}{2}\right]} \right] + 2\sqrt{2} \operatorname{Log} \left[ \cos\left[\frac{x}{2}\right] - \sin\left[\frac{x}{2}\right] \right] - \right. \\ \left. 2\sqrt{2} \operatorname{Log} \left[ \cos\left[\frac{x}{2}\right] + \sin\left[\frac{x}{2}\right] \right] + 2 \operatorname{Log} \left[ \sqrt{2} + 2 \sin[x] \right] - \operatorname{Log} \left[ 2 - \sqrt{2} \cos[x] - \sqrt{2} \sin[x] \right] - \operatorname{Log} \left[ 2 + \sqrt{2} \cos[x] - \sqrt{2} \sin[x] \right] \right)$$

**Problem 389: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \csc[4x] \sin[x]^3 dx$$

Optimal (type 3, 26 leaves, 4 steps):

$$-\frac{1}{4} \operatorname{ArcTanh}[\sin[x]] + \frac{\operatorname{ArcTanh}[\sqrt{2} \sin[x]]}{4\sqrt{2}}$$

Result (type 3, 218 leaves):

$$\frac{1}{16\sqrt{2}} \left( -2i \operatorname{ArcTan} \left[ \frac{\cos\left[\frac{x}{2}\right] - (-1 + \sqrt{2}) \sin\left[\frac{x}{2}\right]}{(1 + \sqrt{2}) \cos\left[\frac{x}{2}\right] - \sin\left[\frac{x}{2}\right]} \right] - 2i \operatorname{ArcTan} \left[ \frac{\cos\left[\frac{x}{2}\right] - (1 + \sqrt{2}) \sin\left[\frac{x}{2}\right]}{(-1 + \sqrt{2}) \cos\left[\frac{x}{2}\right] - \sin\left[\frac{x}{2}\right]} \right] + 4\sqrt{2} \operatorname{Log} \left[ \cos\left[\frac{x}{2}\right] - \sin\left[\frac{x}{2}\right] \right] - \right. \\ \left. 4\sqrt{2} \operatorname{Log} \left[ \cos\left[\frac{x}{2}\right] + \sin\left[\frac{x}{2}\right] \right] + 2 \operatorname{Log} \left[ \sqrt{2} + 2 \sin[x] \right] - \operatorname{Log} \left[ 2 - \sqrt{2} \cos[x] - \sqrt{2} \sin[x] \right] - \operatorname{Log} \left[ 2 + \sqrt{2} \cos[x] - \sqrt{2} \sin[x] \right] \right)$$

**Problem 398: Result more than twice size of optimal antiderivative.**

$$\int \frac{1}{\tan[5x]^{1/3}} dx$$

Optimal (type 3, 57 leaves, 9 steps):

$$-\frac{1}{10} \sqrt{3} \operatorname{ArcTan} \left[ \frac{1 - 2 \tan[5x]^{2/3}}{\sqrt{3}} \right] + \frac{3}{20} \operatorname{Log} \left[ 1 + \tan[5x]^{2/3} \right] - \frac{1}{20} \operatorname{Log} \left[ 1 + \tan[5x]^2 \right]$$

Result (type 3, 121 leaves):

$$\frac{1}{20} \left( -2\sqrt{3} \operatorname{ArcTan} \left[ \sqrt{3} - 2 \tan[5x]^{1/3} \right] - 2\sqrt{3} \operatorname{ArcTan} \left[ \sqrt{3} + 2 \tan[5x]^{1/3} \right] + \right. \\ \left. 2 \operatorname{Log} \left[ 1 + \tan[5x]^{2/3} \right] - \operatorname{Log} \left[ 1 - \sqrt{3} \tan[5x]^{1/3} + \tan[5x]^{2/3} \right] - \operatorname{Log} \left[ 1 + \sqrt{3} \tan[5x]^{1/3} + \tan[5x]^{2/3} \right] \right)$$

**Problem 399: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{1}{(4 + 3 \tan[2x])^{3/2}} dx$$

Optimal (type 3, 87 leaves, 6 steps):

$$-\frac{9 \operatorname{ArcTan}\left[\frac{1-3 \tan[2x]}{\sqrt{2} \sqrt{4+3 \tan[2x]}}\right]}{250 \sqrt{2}} + \frac{13 \operatorname{ArcTanh}\left[\frac{3+\tan[2x]}{\sqrt{2} \sqrt{4+3 \tan[2x]}}\right]}{250 \sqrt{2}} - \frac{3}{25 \sqrt{4+3 \tan[2x]}}$$

Result (type 3, 83 leaves):

$$\frac{(24 - 7i) \sqrt{4 - 3i} \operatorname{ArcTanh}\left[\frac{\sqrt{4+3 \tan[2x]}}{\sqrt{4-3i}}\right] + (24 + 7i) \sqrt{4 + 3i} \operatorname{ArcTanh}\left[\frac{\sqrt{4+3 \tan[2x]}}{\sqrt{4+3i}}\right] - \frac{150}{\sqrt{4+3 \tan[2x]}}}{1250}$$

**Problem 411: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{\cos[x]^3 (\cos[2x] - 3 \tan[x])}{(\sin[x]^2 - \sin[2x]) \sin[2x]^{5/2}} dx$$

Optimal (type 3, 68 leaves, 6 steps):

$$\frac{33}{32} \operatorname{ArcTanh}\left[\frac{1}{2} \sec[x] \sqrt{\sin[2x]}\right] - \frac{9 \cos[x]}{16 \sqrt{\sin[2x]}} - \frac{5 \cos[x] \cot[x]}{24 \sqrt{\sin[2x]}} + \frac{\cos[x] \cot[x]^2}{20 \sqrt{\sin[2x]}}$$

Result (type 4, 150 leaves):

$$\left( \cos[x] \sqrt{\sin[2x]} \left( \frac{1}{15} \csc[x] (-147 - 50 \cot[x] + 12 \csc[x]^2) - \right. \right. \\ \left. \left. 33 \sqrt{\frac{\cos[x]}{-2 + 2 \cos[x]}} \left( \text{EllipticF}\left[\text{ArcSin}\left[\frac{1}{\sqrt{\tan\left[\frac{x}{2}\right]}}\right], -1\right] + \text{EllipticPi}\left[-\frac{2}{-1 + \sqrt{5}}, -\text{ArcSin}\left[\frac{1}{\sqrt{\tan\left[\frac{x}{2}\right]}}\right], -1\right] + \right. \right. \right. \\ \left. \left. \left. \text{EllipticPi}\left[\frac{1}{2}(-1 + \sqrt{5}), -\text{ArcSin}\left[\frac{1}{\sqrt{\tan\left[\frac{x}{2}\right]}}\right], -1\right] \text{Sec}[x] \sqrt{\tan\left[\frac{x}{2}\right]} \right) (\cos[2x] - 3 \tan[x]) \right) \right) / (16 (\cos[x] + \cos[3x] - 6 \sin[x]))$$

**Problem 416: Result unnecessarily involves higher level functions.**

$$\int \frac{\cos[2x] - \sqrt{\sin[2x]}}{\sqrt{\cos[x]^3 \sin[x]}} dx$$

Optimal (type 3, 108 leaves, ? steps):

$$-\sqrt{2} \log[\cos[x] + \sin[x] - \sqrt{2} \sec[x] \sqrt{\cos[x]^3 \sin[x]}] - \\ \frac{\text{ArcSin}[\cos[x] - \sin[x]] \cos[x] \sqrt{\sin[2x]}}{\sqrt{\cos[x]^3 \sin[x]}} - \frac{\text{ArcTanh}[\sin[x]] \cos[x] \sqrt{\sin[2x]}}{\sqrt{\cos[x]^3 \sin[x]}} - \frac{\sin[2x]}{\sqrt{\cos[x]^3 \sin[x]}}$$

Result (type 5, 105 leaves):

$$\left( -4 \cos[x]^3 \text{Hypergeometric2F1}\left[\frac{3}{4}, \frac{3}{4}, \frac{7}{4}, \cos[x]^2\right] \sin[x] - \right. \\ \left. 3 \cos[x] (\sin[x]^2)^{1/4} \left( 2 \sin[x] + \left( -\log\left[\cos\left[\frac{x}{2}\right] - \sin\left[\frac{x}{2}\right]\right] + \log\left[\cos\left[\frac{x}{2}\right] + \sin\left[\frac{x}{2}\right]\right] \right) \sqrt{\sin[2x]} \right) \right) / \left( 3 \sqrt{\cos[x]^3 \sin[x]} (\sin[x]^2)^{1/4} \right)$$

**Problem 417: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{\sqrt{\cos[x] \sin[x]^3} - 2 \sin[2x]}{-\sqrt{\cos[x]^3 \sin[x]} + \sqrt{\tan[x]}} dx$$

Optimal (type 3, 364 leaves, 66 steps):

$$\begin{aligned} & -2\sqrt{2} \operatorname{ArcCoth}\left[\frac{\cos[x] (\cos[x] + \sin[x])}{\sqrt{2} \sqrt{\cos[x]^3 \sin[x]}}\right] + 2^{1/4} \operatorname{ArcCoth}\left[\frac{\cos[x] (\sqrt{2} \cos[x] + \sin[x])}{2^{3/4} \sqrt{\cos[x]^3 \sin[x]}}\right] - 2^{1/4} \operatorname{ArcCoth}\left[\frac{\sqrt{2} + \tan[x]}{2^{3/4} \sqrt{\tan[x]}}\right] - \\ & 2\sqrt{2} \operatorname{ArcTan}\left[\frac{\cos[x] (\cos[x] - \sin[x])}{\sqrt{2} \sqrt{\cos[x]^3 \sin[x]}}\right] + 2^{1/4} \operatorname{ArcTan}\left[\frac{\cos[x] (\sqrt{2} \cos[x] - \sin[x])}{2^{3/4} \sqrt{\cos[x]^3 \sin[x]}}\right] - 2^{1/4} \operatorname{ArcTan}\left[\frac{\sqrt{2} - \tan[x]}{2^{3/4} \sqrt{\tan[x]}}\right] + \\ & 4 \operatorname{Csc}[x] \operatorname{Sec}[x] \sqrt{\cos[x]^3 \sin[x]} + \frac{1}{4} \operatorname{Csc}[x]^2 \operatorname{Log}[1 + \cos[x]^2] \operatorname{Sec}[x]^2 \sqrt{\cos[x]^3 \sin[x]} \sqrt{\cos[x] \sin[x]^3} + \\ & \frac{1}{2} \operatorname{Csc}[x]^2 \operatorname{Log}[\sin[x]] \operatorname{Sec}[x]^2 \sqrt{\cos[x]^3 \sin[x]} \sqrt{\cos[x] \sin[x]^3} + \frac{4}{\sqrt{\tan[x]}} - \\ & \frac{1}{4} \operatorname{Csc}[x]^2 \operatorname{Log}[1 + \cos[x]^2] \sqrt{\cos[x] \sin[x]^3} \sqrt{\tan[x]} + \frac{1}{2} \operatorname{Csc}[x]^2 \operatorname{Log}[\sin[x]] \sqrt{\cos[x] \sin[x]^3} \sqrt{\tan[x]} \end{aligned}$$

Result (type 5, 2057 leaves):

$$\begin{aligned} & \frac{\cos[x] \operatorname{Csc}\left[\frac{x}{2}\right] \left(4 \operatorname{Log}\left[\operatorname{Sec}\left[\frac{x}{2}\right]^2\right] - 2 \operatorname{Log}\left[\operatorname{Tan}\left[\frac{x}{2}\right]\right] - \operatorname{Log}\left[1 + \operatorname{Tan}\left[\frac{x}{2}\right]^4\right]\right) \operatorname{Sec}\left[\frac{x}{2}\right] \sqrt{\cos[x] \sin[x]^3}}{8 \sqrt{\cos[x]^3 \sin[x]}} + \\ & \left( (1+i) \left( (4+4i) \operatorname{EllipticPi}\left[-i, -\operatorname{ArcSin}\left[\sqrt{\operatorname{Tan}\left[\frac{x}{2}\right]}\right], -1\right] - (4+4i) \operatorname{EllipticPi}\left[i, -\operatorname{ArcSin}\left[\sqrt{\operatorname{Tan}\left[\frac{x}{2}\right]}\right], -1\right] + \right. \right. \\ & \quad (-1)^{1/4} \left( -\operatorname{EllipticPi}\left[-(-1)^{1/4}, -\operatorname{ArcSin}\left[\sqrt{\operatorname{Tan}\left[\frac{x}{2}\right]}\right], -1\right] + \operatorname{EllipticPi}\left[(-1)^{1/4}, -\operatorname{ArcSin}\left[\sqrt{\operatorname{Tan}\left[\frac{x}{2}\right]}\right], -1\right] - \right. \\ & \quad \left. \left. \operatorname{EllipticPi}\left[-(-1)^{3/4}, -\operatorname{ArcSin}\left[\sqrt{\operatorname{Tan}\left[\frac{x}{2}\right]}\right], -1\right] + \operatorname{EllipticPi}\left[(-1)^{3/4}, -\operatorname{ArcSin}\left[\sqrt{\operatorname{Tan}\left[\frac{x}{2}\right]}\right], -1\right] \right) \right) \\ & \operatorname{Sec}\left[\frac{x}{2}\right]^4 \sqrt{\cos[x]^3 \sin[x]} \left( \frac{2\sqrt{2} \operatorname{Sec}[x]^2 \sqrt{2 \sin[2x] + \sin[4x]}}{3 + \cos[2x]} + \frac{\sqrt{2} \cos[2x] \operatorname{Sec}[x]^2 \sqrt{2 \sin[2x] + \sin[4x]}}{3 + \cos[2x]} \right) \Big/ \end{aligned}$$

$$\begin{aligned}
& \left( \sqrt{\cos[x] \sec\left[\frac{x}{2}\right]^2} \sqrt{\tan\left[\frac{x}{2}\right]} \left(-1 + \tan\left[\frac{x}{2}\right]^2\right) \left( -\frac{1}{\sqrt{\cos[x] \sec\left[\frac{x}{2}\right]^2} \left(-1 + \tan\left[\frac{x}{2}\right]^2\right)^2} \right. \right. \\
& (1+i) \left( (4+4i) \operatorname{EllipticPi}[-i, -\operatorname{ArcSin}\left[\sqrt{\tan\left[\frac{x}{2}\right]}\right], -1\right] - (4+4i) \operatorname{EllipticPi}[i, -\operatorname{ArcSin}\left[\sqrt{\tan\left[\frac{x}{2}\right]}\right], -1\right] + \\
& (-1)^{1/4} \left( -\operatorname{EllipticPi}[-(-1)^{1/4}, -\operatorname{ArcSin}\left[\sqrt{\tan\left[\frac{x}{2}\right]}\right], -1\right] + \operatorname{EllipticPi}[(-1)^{1/4}, -\operatorname{ArcSin}\left[\sqrt{\tan\left[\frac{x}{2}\right]}\right], -1\right] - \\
& \left. \left. \operatorname{EllipticPi}[-(-1)^{3/4}, -\operatorname{ArcSin}\left[\sqrt{\tan\left[\frac{x}{2}\right]}\right], -1\right] + \operatorname{EllipticPi}[(-1)^{3/4}, -\operatorname{ArcSin}\left[\sqrt{\tan\left[\frac{x}{2}\right]}\right], -1\right] \right) \right) \\
& \sec\left[\frac{x}{2}\right]^6 \sqrt{\cos[x]^3 \sin[x]} \sqrt{\tan\left[\frac{x}{2}\right]} - \frac{1}{\sqrt{\cos[x] \sec\left[\frac{x}{2}\right]^2} \tan\left[\frac{x}{2}\right]^{3/2} \left(-1 + \tan\left[\frac{x}{2}\right]^2\right)} \\
& \left( \frac{1}{4} + \frac{i}{4} \right) \left( (4+4i) \operatorname{EllipticPi}[-i, -\operatorname{ArcSin}\left[\sqrt{\tan\left[\frac{x}{2}\right]}\right], -1\right] - (4+4i) \operatorname{EllipticPi}[i, -\operatorname{ArcSin}\left[\sqrt{\tan\left[\frac{x}{2}\right]}\right], -1\right] + \\
& (-1)^{1/4} \left( -\operatorname{EllipticPi}[-(-1)^{1/4}, -\operatorname{ArcSin}\left[\sqrt{\tan\left[\frac{x}{2}\right]}\right], -1\right] + \operatorname{EllipticPi}[(-1)^{1/4}, -\operatorname{ArcSin}\left[\sqrt{\tan\left[\frac{x}{2}\right]}\right], -1\right] - \\
& \left. \left. \operatorname{EllipticPi}[-(-1)^{3/4}, -\operatorname{ArcSin}\left[\sqrt{\tan\left[\frac{x}{2}\right]}\right], -1\right] + \operatorname{EllipticPi}[(-1)^{3/4}, -\operatorname{ArcSin}\left[\sqrt{\tan\left[\frac{x}{2}\right]}\right], -1\right] \right) \right) \\
& \sec\left[\frac{x}{2}\right]^6 \sqrt{\cos[x]^3 \sin[x]} + \frac{1}{\sqrt{\cos[x] \sec\left[\frac{x}{2}\right]^2} \sqrt{\cos[x]^3 \sin[x]} \sqrt{\tan\left[\frac{x}{2}\right]} \left(-1 + \tan\left[\frac{x}{2}\right]^2\right)} \\
& \left( \frac{1}{2} + \frac{i}{2} \right) \left( (4+4i) \operatorname{EllipticPi}[-i, -\operatorname{ArcSin}\left[\sqrt{\tan\left[\frac{x}{2}\right]}\right], -1\right] - (4+4i) \operatorname{EllipticPi}[i, -\operatorname{ArcSin}\left[\sqrt{\tan\left[\frac{x}{2}\right]}\right], -1\right] + \\
& (-1)^{1/4} \left( -\operatorname{EllipticPi}[-(-1)^{1/4}, -\operatorname{ArcSin}\left[\sqrt{\tan\left[\frac{x}{2}\right]}\right], -1\right] + \operatorname{EllipticPi}[(-1)^{1/4}, -\operatorname{ArcSin}\left[\sqrt{\tan\left[\frac{x}{2}\right]}\right], -1\right] - \\
& \left. \left. \operatorname{EllipticPi}[-(-1)^{3/4}, -\operatorname{ArcSin}\left[\sqrt{\tan\left[\frac{x}{2}\right]}\right], -1\right] + \operatorname{EllipticPi}[(-1)^{3/4}, -\operatorname{ArcSin}\left[\sqrt{\tan\left[\frac{x}{2}\right]}\right], -1\right] \right) \right) \\
& \sec\left[\frac{x}{2}\right]^4 \left( \cos[x]^4 - 3 \cos[x]^2 \sin[x]^2 \right) + \frac{1}{\sqrt{\cos[x] \sec\left[\frac{x}{2}\right]^2} \left(-1 + \tan\left[\frac{x}{2}\right]^2\right)}
\end{aligned}$$



$$(2 + 2i) \left( (4 + 4i) \operatorname{EllipticPi}[-i, -\operatorname{ArcSin}[\sqrt{\tan[\frac{x}{2}]}], -1] - (4 + 4i) \operatorname{EllipticPi}[i, -\operatorname{ArcSin}[\sqrt{\tan[\frac{x}{2}]}], -1] + \right. \\ \left. (-1)^{1/4} \left( -\operatorname{EllipticPi}[-(-1)^{1/4}, -\operatorname{ArcSin}[\sqrt{\tan[\frac{x}{2}]}], -1] + \operatorname{EllipticPi}[(-1)^{1/4}, -\operatorname{ArcSin}[\sqrt{\tan[\frac{x}{2}]}], -1] - \right. \right. \\ \left. \left. \operatorname{EllipticPi}[-(-1)^{3/4}, -\operatorname{ArcSin}[\sqrt{\tan[\frac{x}{2}]}], -1] + \operatorname{EllipticPi}[(-1)^{3/4}, -\operatorname{ArcSin}[\sqrt{\tan[\frac{x}{2}]}], -1] \right) \right)$$

$$\operatorname{Sec}[\frac{x}{2}]^4 \sqrt{\cos[x]^3 \sin[x]} \sqrt{\tan[\frac{x}{2}]} - \frac{1}{(\cos[x] \operatorname{Sec}[\frac{x}{2}]^2)^{3/2} \sqrt{\tan[\frac{x}{2}]} (-1 + \tan[\frac{x}{2}])}$$

$$\left(\frac{1}{2} + \frac{i}{2}\right) \left( (4 + 4i) \operatorname{EllipticPi}[-i, -\operatorname{ArcSin}[\sqrt{\tan[\frac{x}{2}]}], -1] - (4 + 4i) \operatorname{EllipticPi}[i, -\operatorname{ArcSin}[\sqrt{\tan[\frac{x}{2}]}], -1] + \right. \\ \left. (-1)^{1/4} \left( -\operatorname{EllipticPi}[-(-1)^{1/4}, -\operatorname{ArcSin}[\sqrt{\tan[\frac{x}{2}]}], -1] + \operatorname{EllipticPi}[(-1)^{1/4}, -\operatorname{ArcSin}[\sqrt{\tan[\frac{x}{2}]}], -1] - \right. \right. \\ \left. \left. \operatorname{EllipticPi}[-(-1)^{3/4}, -\operatorname{ArcSin}[\sqrt{\tan[\frac{x}{2}]}], -1] + \operatorname{EllipticPi}[(-1)^{3/4}, -\operatorname{ArcSin}[\sqrt{\tan[\frac{x}{2}]}], -1] \right) \right) \operatorname{Sec}[\frac{x}{2}]^4$$

$$\sqrt{\cos[x]^3 \sin[x]} \left( -\operatorname{Sec}[\frac{x}{2}]^2 \sin[x] + \cos[x] \operatorname{Sec}[\frac{x}{2}]^2 \tan[\frac{x}{2}] \right) + \frac{1}{\sqrt{\cos[x] \operatorname{Sec}[\frac{x}{2}]^2} \sqrt{\tan[\frac{x}{2}]} (-1 + \tan[\frac{x}{2}])^2}$$

$$(1 + i) \operatorname{Sec}[\frac{x}{2}]^4 \sqrt{\cos[x]^3 \sin[x]} \left( \frac{(1 + i) \operatorname{Sec}[\frac{x}{2}]^2}{\sqrt{1 - \tan[\frac{x}{2}]} (1 - i \tan[\frac{x}{2}]) \sqrt{\tan[\frac{x}{2}]} \sqrt{1 + \tan[\frac{x}{2}]}} - \right.$$

$$\left. \frac{(1 + i) \operatorname{Sec}[\frac{x}{2}]^2}{\sqrt{1 - \tan[\frac{x}{2}]} (1 + i \tan[\frac{x}{2}]) \sqrt{\tan[\frac{x}{2}]} \sqrt{1 + \tan[\frac{x}{2}]}} + (-1)^{1/4} \left( -\frac{\operatorname{Sec}[\frac{x}{2}]^2}{4 \sqrt{1 - \tan[\frac{x}{2}]} \sqrt{\tan[\frac{x}{2}]} \sqrt{1 + \tan[\frac{x}{2}]} (1 - (-1)^{1/4} \tan[\frac{x}{2}])} + \right.$$

$$\left. \frac{\operatorname{Sec}[\frac{x}{2}]^2}{4 \sqrt{1 - \tan[\frac{x}{2}]} \sqrt{\tan[\frac{x}{2}]} \sqrt{1 + \tan[\frac{x}{2}]} (1 + (-1)^{1/4} \tan[\frac{x}{2}])} - \frac{\operatorname{Sec}[\frac{x}{2}]^2}{4 \sqrt{1 - \tan[\frac{x}{2}]} \sqrt{\tan[\frac{x}{2}]} \sqrt{1 + \tan[\frac{x}{2}]} (1 - (-1)^{3/4} \tan[\frac{x}{2}])} + \right.$$

$$\frac{\frac{\text{Sec}\left[\frac{x}{2}\right]^2}{4\sqrt{1-\text{Tan}\left[\frac{x}{2}\right]}\sqrt{\text{Tan}\left[\frac{x}{2}\right]}\sqrt{1+\text{Tan}\left[\frac{x}{2}\right]}\left(1+(-1)^{3/4}\text{Tan}\left[\frac{x}{2}\right]\right)}}{\sqrt{\text{Tan}[x]}} + \frac{2(\text{Cos}[x]^2)^{3/4}\text{Hypergeometric2F1}\left[\frac{3}{4}, \frac{3}{4}, \frac{7}{4}, \frac{\text{Sin}[x]^2}{2\left(1-\frac{\text{Sin}[x]^2}{2}\right)}\right](2-\text{Sin}[x]^2)\text{Tan}[x]^{3/2}}{3\left(1-\frac{\text{Sin}[x]^2}{2}\right)^{3/4}(-2+\text{Sin}[x]^2)} +$$

$$\frac{\sqrt{2\text{Sin}[2x] + \text{Sin}[4x]}}{\left(\sqrt{2}\text{Cot}[x] + \sqrt{2}\text{Tan}[x]\right) + \left(\text{Csc}[x]^2\left(4\text{Log}\left[\sqrt{\text{Tan}[x]}\right] - \text{Log}\left[2 + \text{Tan}[x]^2\right]\right)\right)} +$$

$$\frac{\frac{\text{Sec}[x]^2\sqrt{2\text{Sin}[2x] - \text{Sin}[4x]}}{\sqrt{\text{Tan}[x]}\left(2 + \text{Tan}[x]^2\right)}\left(4\sqrt{2}\left(3 + \text{Cos}[2x]\right)\left(1 + \text{Tan}[x]^2\right)^2\right)}{\sqrt{2}\left(3 + \text{Cos}[2x]\right)\left(1 + \text{Tan}[x]^2\right)^2}$$

**Problem 424: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{\text{Sin}[5x]}{(5\text{Cos}[x]^2 + 9\text{Sin}[x]^2)^{5/2}} dx$$

Optimal (type 3, 48 leaves, 4 steps):

$$-\frac{1}{2}\text{ArcSin}\left[\frac{2\text{Cos}[x]}{3}\right] - \frac{55\text{Cos}[x]}{27(9-4\text{Cos}[x]^2)^{3/2}} + \frac{295\text{Cos}[x]}{243\sqrt{9-4\text{Cos}[x]^2}}$$

Result (type 3, 63 leaves):

$$\frac{2550\text{Cos}[x] - 590\text{Cos}[3x] + 243\sqrt{7-2\text{Cos}[2x]}^{3/2}\text{Log}\left[2\sqrt{7-2\text{Cos}[2x]} + \sqrt{7-2\text{Cos}[2x]}\right]}{486(7-2\text{Cos}[2x])^{3/2}}$$

**Problem 426: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{\operatorname{Csc}[x]^2 (-2 \operatorname{Cos}[x]^3 (-1 + \operatorname{Sin}[x]) + \operatorname{Cos}[2x] \operatorname{Sin}[x])}{\sqrt{-5 + \operatorname{Sin}[x]^2}} dx$$

Optimal (type 3, 111 leaves, 18 steps):

$$2 \operatorname{ArcTan}\left[\frac{\operatorname{Cos}[x]}{\sqrt{-5 + \operatorname{Sin}[x]^2}}\right] - \frac{\operatorname{ArcTan}\left[\frac{\sqrt{5} \operatorname{Cos}[x]}{\sqrt{-5 + \operatorname{Sin}[x]^2}}\right]}{\sqrt{5}} - \frac{2 \operatorname{ArcTan}\left[\frac{\sqrt{-5 + \operatorname{Sin}[x]^2}}{\sqrt{5}}\right]}{\sqrt{5}} -$$

$$2 \operatorname{ArcTanh}\left[\frac{\operatorname{Sin}[x]}{\sqrt{-5 + \operatorname{Sin}[x]^2}}\right] + 2 \sqrt{-5 + \operatorname{Sin}[x]^2} + \frac{2}{5} \operatorname{Csc}[x] \sqrt{-5 + \operatorname{Sin}[x]^2}$$

Result (type 4, 338 leaves):

$$\frac{1}{25 \sqrt{2} \sqrt{-9 - \operatorname{Cos}[2x]}}$$

$$\left( (16 - 32i) \sqrt{5} \operatorname{Cos}\left[\frac{x}{2}\right]^2 \sqrt{\frac{(1 + 2i)(-2i + \operatorname{Cos}[x])}{1 + \operatorname{Cos}[x]}} \sqrt{\frac{(1 - 2i)(2i + \operatorname{Cos}[x])}{1 + \operatorname{Cos}[x]}} \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{(1 + 2i) \operatorname{Tan}\left[\frac{x}{2}\right]}{\sqrt{5}}\right], -\frac{7}{25} + \frac{24i}{25}\right] - \right.$$

$$(32 - 64i) \sqrt{5} \operatorname{Cos}\left[\frac{x}{2}\right]^2 \sqrt{\frac{(1 + 2i)(-2i + \operatorname{Cos}[x])}{1 + \operatorname{Cos}[x]}} \sqrt{\frac{(1 - 2i)(2i + \operatorname{Cos}[x])}{1 + \operatorname{Cos}[x]}}$$

$$\operatorname{EllipticPi}\left[\frac{3}{5} + \frac{4i}{5}, \operatorname{ArcSin}\left[\frac{(1 + 2i) \operatorname{Tan}\left[\frac{x}{2}\right]}{\sqrt{5}}\right], -\frac{7}{25} + \frac{24i}{25}\right] -$$

$$5 \left( 85 + \sqrt{10} \operatorname{ArcTan}\left[\frac{\sqrt{10} \operatorname{Cos}[x]}{\sqrt{-9 - \operatorname{Cos}[2x]}}\right] \sqrt{-9 - \operatorname{Cos}[2x]} + 2 \sqrt{10} \operatorname{ArcTan}\left[\frac{\sqrt{-9 - \operatorname{Cos}[2x]}}{\sqrt{10}}\right] \sqrt{-9 - \operatorname{Cos}[2x]} + 18 \operatorname{Csc}[x] + \right.$$

$$\left. \left. 2 \operatorname{Cos}[2x] \operatorname{Csc}[x] + 10i \sqrt{2} \sqrt{-9 - \operatorname{Cos}[2x]} \operatorname{Log}\left[i \sqrt{2} \operatorname{Cos}[x] + \sqrt{-9 - \operatorname{Cos}[2x]}\right] + 5 \operatorname{Csc}[x] \operatorname{Sin}[3x] \right) \right)$$

**Problem 427: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{\operatorname{Cos}[3x]}{-\sqrt{-1 + 8 \operatorname{Cos}[x]^2} + \sqrt{3 \operatorname{Cos}[x]^2 - \operatorname{Sin}[x]^2}} dx$$

Optimal (type 3, 112 leaves, 27 steps):

$$\frac{5 \operatorname{ArcSin}\left[2 \sqrt{\frac{2}{7}} \sin[x]\right]}{4 \sqrt{2}} + \frac{3}{4} \operatorname{ArcSin}\left[\frac{2 \sin[x]}{\sqrt{3}}\right] - \frac{3}{4} \operatorname{ArcTan}\left[\frac{\sin[x]}{\sqrt{-1+4 \cos[x]^2}}\right] -$$

$$\frac{3}{4} \operatorname{ArcTan}\left[\frac{\sin[x]}{\sqrt{-1+8 \cos[x]^2}}\right] - \frac{1}{2} \sqrt{-1+4 \cos[x]^2} \sin[x] - \frac{1}{2} \sqrt{-1+8 \cos[x]^2} \sin[x]$$

Result (type 3, 131 leaves):

$$\frac{1}{8} \left( -6 \operatorname{ArcTan}\left[\frac{\sin[x]}{\sqrt{1+2 \cos[2x]}}\right] - 6 \operatorname{ArcTan}\left[\frac{\sin[x]}{\sqrt{3+4 \cos[2x]}}\right] - 6 i \operatorname{Log}\left[\sqrt{1+2 \cos[2x]} + 2 i \sin[x]\right] -$$

$$5 i \sqrt{2} \operatorname{Log}\left[\sqrt{3+4 \cos[2x]} + 2 i \sqrt{2} \sin[x]\right] - 4 \sqrt{1+2 \cos[2x]} \sin[x] - 4 \sqrt{3+4 \cos[2x]} \sin[x] \right)$$

**Problem 434: Result unnecessarily involves imaginary or complex numbers.**

$$\int (4 - 5 \sec[x]^2)^{3/2} dx$$

Optimal (type 3, 68 leaves, 7 steps):

$$8 \operatorname{ArcTan}\left[\frac{2 \tan[x]}{\sqrt{-1-5 \tan[x]^2}}\right] - \frac{7}{2} \sqrt{5} \operatorname{ArcTan}\left[\frac{\sqrt{5} \tan[x]}{\sqrt{-1-5 \tan[x]^2}}\right] - \frac{5}{2} \tan[x] \sqrt{-1-5 \tan[x]^2}$$

Result (type 3, 115 leaves):

$$-\frac{1}{2 (-3 + 2 \cos[2x])^{3/2}} (-5 + 4 \cos[x]^2) \sec[x] \sqrt{4 - 5 \sec[x]^2}$$

$$\left( 7 \sqrt{5} \operatorname{ArcTan}\left[\frac{\sqrt{5} \sin[x]}{\sqrt{-3 + 2 \cos[2x]}}\right] \cos[x]^2 + 16 i \cos[x]^2 \operatorname{Log}\left[\sqrt{-3 + 2 \cos[2x]} + 2 i \sin[x]\right] + 5 \sqrt{-3 + 2 \cos[2x]} \sin[x] \right)$$

**Problem 438: Result more than twice size of optimal antiderivative.**

$$\int \frac{(3 + \sin[x]^2) \tan[x]^3}{(-2 + \cos[x]^2) (5 - 4 \sec[x]^2)^{3/2}} dx$$

Optimal (type 3, 73 leaves, 16 steps):

$$-\frac{\text{ArcTanh}\left[\frac{\sqrt{5-4\text{Sec}[x]^2}}{\sqrt{3}}\right]}{6\sqrt{3}} - \frac{\text{ArcTanh}\left[\frac{\sqrt{5-4\text{Sec}[x]^2}}{\sqrt{5}}\right]}{5\sqrt{5}} - \frac{2}{15\sqrt{5-4\text{Sec}[x]^2}}$$

Result (type 3, 234 leaves):

$$\frac{1}{60(5-4\text{Sec}[x]^2)^{3/2}} \text{Sec}[x]^2 \left( 12 - 20 \text{Cos}[2x] + \left( \sqrt{2} (-3 + 5 \text{Cos}[2x]) \right)^{3/2} \left( 15 \sqrt{3} \text{ArcTanh}\left[\frac{\sqrt{-3 + 5 \text{Cos}[2x]}}{\sqrt{6} \sqrt{\text{Cos}[x]^2}}\right] \text{Sin}[x]^2 - \right. \right. \\ \left. \left. 18 \sqrt{5} \left( \text{Log}[10 \text{Sin}[x]^2] - \text{Log}\left[5 \left( -\sqrt{-3 + 5 \text{Cos}[2x]} + \text{Cos}[2x] \sqrt{-3 + 5 \text{Cos}[2x]} + \sqrt{10} \sqrt{\text{Sin}[x]^2} \sqrt{\text{Sin}[2x]^2} \right)\right] \right) \text{Sin}[x]^2 - \right. \right. \\ \left. \left. 20 \sqrt{3} \text{ArcTanh}\left[\frac{\sqrt{6} \text{Cos}[x]}{\sqrt{-3 + 5 \text{Cos}[2x]}}\right] \text{Sec}[x] \sqrt{\text{Sin}[x]^2} \sqrt{\text{Sin}[2x]^2} \right) \right) / \left( 15 \sqrt{\text{Sin}[x]^2} \sqrt{\text{Sin}[2x]^2} \right)$$

**Problem 439: Result more than twice size of optimal antiderivative.**

$$\int \frac{\text{Csc}[x]^2 \left( \text{Sec}[x]^2 - 3 \text{Tan}[x] \sqrt{4 \text{Sec}[x]^2 + 5 \text{Tan}[x]^2} \right)}{(4 \text{Sec}[x]^2 + 5 \text{Tan}[x]^2)^{3/2}} dx$$

Optimal (type 3, 57 leaves, 10 steps):

$$-\frac{3}{4} \text{Log}[\text{Tan}[x]] + \frac{3}{8} \text{Log}[4 + 9 \text{Tan}[x]^2] - \frac{\text{Cot}[x]}{4 \sqrt{4 + 9 \text{Tan}[x]^2}} - \frac{7 \text{Tan}[x]}{8 \sqrt{4 + 9 \text{Tan}[x]^2}}$$

Result (type 3, 116 leaves):

$$\frac{1}{16 \sqrt{\frac{13-5 \text{Cos}[2x]}{1+\text{Cos}[2x]}}} \left( 5 \text{Cot}[x] + 6 \sqrt{\frac{13-5 \text{Cos}[2x]}{1+\text{Cos}[2x]}} \text{Log}\left[1 + 7 \text{Tan}\left[\frac{x}{2}\right]^2 + \text{Tan}\left[\frac{x}{2}\right]^4\right] - 9 \text{Csc}[x] \text{Sec}[x] - 5 \text{Tan}[x] - 6 \sqrt{2} \text{Log}\left[\text{Tan}\left[\frac{x}{2}\right]\right] \sqrt{-5 + 13 \text{Sec}[x]^2 + 5 \text{Tan}[x]^2} \right)$$

**Problem 442: Result unnecessarily involves higher level functions.**

$$\int \frac{\text{Tan}[x]}{(a^3 + b^3 \text{Tan}[x]^2)^{1/3}} dx$$

Optimal (type 3, 133 leaves, 6 steps):

$$\frac{\sqrt{3} \operatorname{ArcTan}\left[\frac{1 + 2(a^3 + b^3 \tan[x]^2)^{1/3}}{(a^3 - b^3)^{1/3}}\right]}{2(a^3 - b^3)^{1/3}} + \frac{\operatorname{Log}[\operatorname{Cos}[x]]}{2(a^3 - b^3)^{1/3}} + \frac{3 \operatorname{Log}\left[(a^3 - b^3)^{1/3} - (a^3 + b^3 \tan[x]^2)^{1/3}\right]}{4(a^3 - b^3)^{1/3}}$$

Result (type 5, 90 leaves):

$$\frac{3 \left(\frac{a^3 + b^3 + (a^3 - b^3) \operatorname{Cos}[2x]}{b^3}\right)^{1/3} \operatorname{Hypergeometric2F1}\left[\frac{1}{3}, \frac{1}{3}, \frac{4}{3}, \frac{(-a^3 + b^3) \operatorname{Cos}[x]^2}{b^3}\right]}{2 \left((a^3 + b^3 + (a^3 - b^3) \operatorname{Cos}[2x]) \operatorname{Sec}[x]^2\right)^{1/3}}$$

**Problem 443: Result unnecessarily involves higher level functions.**

$$\int \tan[x] (1 - 7 \tan[x]^2)^{2/3} dx$$

Optimal (type 3, 69 leaves, 7 steps):

$$2\sqrt{3} \operatorname{ArcTan}\left[\frac{1 + (1 - 7 \tan[x]^2)^{1/3}}{\sqrt{3}}\right] + 2 \operatorname{Log}[\operatorname{Cos}[x]] + 3 \operatorname{Log}\left[2 - (1 - 7 \tan[x]^2)^{1/3}\right] + \frac{3}{4} (1 - 7 \tan[x]^2)^{2/3}$$

Result (type 5, 42 leaves):

$$-\frac{3}{4} \left(-1 + \operatorname{Hypergeometric2F1}\left[\frac{2}{3}, 1, \frac{5}{3}, \frac{1}{8}(-3 + 4 \operatorname{Cos}[2x]) \operatorname{Sec}[x]^2\right]\right) (1 - 7 \tan[x]^2)^{2/3}$$

**Problem 444: Result unnecessarily involves higher level functions.**

$$\int \frac{\operatorname{Cot}[x]}{(a^4 + b^4 \operatorname{Csc}[x]^2)^{1/4}} dx$$

Optimal (type 3, 52 leaves, 6 steps):

$$-\frac{\operatorname{ArcTan}\left[\frac{(a^4 + b^4 \operatorname{Csc}[x]^2)^{1/4}}{a}\right]}{a} + \frac{\operatorname{ArcTanh}\left[\frac{(a^4 + b^4 \operatorname{Csc}[x]^2)^{1/4}}{a}\right]}{a}$$

Result (type 5, 84 leaves):

$$-\frac{(-a^4 - 2b^4 + a^4 \operatorname{Cos}[2x]) \operatorname{Csc}[x]^2 \operatorname{Hypergeometric2F1}\left[\frac{3}{4}, 1, \frac{7}{4}, -\frac{(-a^4 - 2b^4 + a^4 \operatorname{Cos}[2x]) \operatorname{Csc}[x]^2}{2a^4}\right]}{3a^4 (a^4 + b^4 \operatorname{Csc}[x]^2)^{1/4}}$$

**Problem 445: Result unnecessarily involves higher level functions.**

$$\int \frac{\text{Cot}[x]}{(a^4 - b^4 \text{Csc}[x]^2)^{1/4}} dx$$

Optimal (type 3, 54 leaves, 6 steps):

$$-\frac{\text{ArcTan}\left[\frac{(a^4 - b^4 \text{Csc}[x]^2)^{1/4}}{a}\right]}{a} + \frac{\text{ArcTanh}\left[\frac{(a^4 - b^4 \text{Csc}[x]^2)^{1/4}}{a}\right]}{a}$$

Result (type 5, 85 leaves):

$$-\frac{(-a^4 + 2b^4 + a^4 \cos[2x]) \text{Csc}[x]^2 \text{Hypergeometric2F1}\left[\frac{3}{4}, 1, \frac{7}{4}, -\frac{(-a^4 + 2b^4 + a^4 \cos[2x]) \text{Csc}[x]^2}{2a^4}\right]}{3a^4 (a^4 - b^4 \text{Csc}[x]^2)^{1/4}}$$

**Problem 446: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{\text{Sec}[x]^2 \text{Tan}[x] \left( (1 - 3 \text{Sec}[x]^2)^{1/3} \text{Sin}[x]^2 + 3 \text{Tan}[x]^2 \right)}{(1 - 3 \text{Sec}[x]^2)^{5/6} \left( 1 - \sqrt{1 - 3 \text{Sec}[x]^2} \right)} dx$$

Optimal (type 3, 133 leaves, 29 steps):

$$\sqrt{3} \text{ArcTan}\left[\frac{1 + 2(1 - 3 \text{Sec}[x]^2)^{1/6}}{\sqrt{3}}\right] + \frac{1}{4} \text{Log}[\text{Sec}[x]^2] - \frac{3}{2} \text{Log}[1 - (1 - 3 \text{Sec}[x]^2)^{1/6}] +$$

$$\frac{1}{3} \text{Log}[1 - \sqrt{1 - 3 \text{Sec}[x]^2}] - (1 - 3 \text{Sec}[x]^2)^{1/6} - \frac{1}{4} (1 - 3 \text{Sec}[x]^2)^{2/3} + \frac{1}{2(1 - \sqrt{1 - 3 \text{Sec}[x]^2})}$$

Result (type 6, 4397 leaves):

$$-\left( \left( 3 \left( 6 + \left( \frac{-5 + \cos[2x]}{1 + \cos[2x]} \right)^{1/3} + \cos[2x] \left( \frac{-5 + \cos[2x]}{1 + \cos[2x]} \right)^{1/3} \right) \left( 3 \text{Sec}[x]^2 + (1 - 3 \text{Sec}[x]^2)^{1/3} \right) \right.$$

$$\left. \text{Sin}[x]^2 \text{Tan}[x] (-2 - 3 \text{Tan}[x]^2)^{5/6} (1 + \text{Tan}[x]^2) (2 + 3 \text{Tan}[x]^2) \left( -8 \text{AppellF1}\left[1, \frac{1}{2}, 1, 2, -\frac{3}{2} \text{Tan}[x]^2, -\text{Tan}[x]^2\right] + \right. \right.$$

$$\left. \left. 4 \text{AppellF1}\left[2, \frac{1}{2}, 2, 3, -\frac{3}{2} \text{Tan}[x]^2, -\text{Tan}[x]^2\right] \text{Tan}[x]^2 + 3 \text{AppellF1}\left[2, \frac{3}{2}, 1, 3, -\frac{3}{2} \text{Tan}[x]^2, -\text{Tan}[x]^2\right] \text{Tan}[x]^2 \right)^2 \right)$$

$$\begin{aligned}
& \left( \left( 4 \operatorname{AppellF1} \left[ 2, \frac{1}{2}, 2, 3, -\frac{3}{2} \tan[x]^2, -\tan[x]^2 \right] + 3 \operatorname{AppellF1} \left[ 2, \frac{3}{2}, 1, 3, -\frac{3}{2} \tan[x]^2, -\tan[x]^2 \right] \right) \tan[x]^2 \right. \\
& \quad \left( 30 \times 3^{2/3} \operatorname{Hypergeometric2F1} \left[ \frac{1}{3}, \frac{1}{3}, \frac{4}{3}, \frac{1}{3+3 \tan[x]^2} \right] \sqrt{-2-3 \tan[x]^2} (1+\tan[x]^2) \left( \frac{2+3 \tan[x]^2}{1+\tan[x]^2} \right)^{1/3} + \right. \\
& \quad \left. 12 \times 3^{1/6} \operatorname{Hypergeometric2F1} \left[ \frac{5}{6}, \frac{5}{6}, \frac{11}{6}, \frac{1}{3+3 \tan[x]^2} \right] (1+\tan[x]^2) \left( \frac{2+3 \tan[x]^2}{1+\tan[x]^2} \right)^{5/6} + \right. \\
& \quad \left. 5 \left( 2 \operatorname{Log} [1+\tan[x]^2] (-2-3 \tan[x]^2)^{5/6} (1+\tan[x]^2) + 9 \tan[x]^4 \left( 4 + \sqrt{-2-3 \tan[x]^2} \right) + 3 \tan[x]^2 \right. \right. \\
& \quad \left. \left. \left( 20 - 2 (-2-3 \tan[x]^2)^{1/3} + 5 \sqrt{-2-3 \tan[x]^2} \right) + 2 \left( 12 - 2 (-2-3 \tan[x]^2)^{1/3} + 3 \sqrt{-2-3 \tan[x]^2} + (-2-3 \tan[x]^2)^{5/6} \right) \right) \right) - \\
& \quad 8 \operatorname{AppellF1} \left[ 1, \frac{1}{2}, 1, 2, -\frac{3}{2} \tan[x]^2, -\tan[x]^2 \right] \left( 30 \times 3^{2/3} \operatorname{Hypergeometric2F1} \left[ \frac{1}{3}, \frac{1}{3}, \frac{4}{3}, \frac{1}{3+3 \tan[x]^2} \right] \sqrt{-2-3 \tan[x]^2} \right. \\
& \quad \left. (1+\tan[x]^2) \left( \frac{2+3 \tan[x]^2}{1+\tan[x]^2} \right)^{1/3} + 12 \times 3^{1/6} \operatorname{Hypergeometric2F1} \left[ \frac{5}{6}, \frac{5}{6}, \frac{11}{6}, \frac{1}{3+3 \tan[x]^2} \right] (1+\tan[x]^2) \left( \frac{2+3 \tan[x]^2}{1+\tan[x]^2} \right)^{5/6} + \right. \\
& \quad \left. 5 \left( 2 \operatorname{Log} [1+\tan[x]^2] (-2-3 \tan[x]^2)^{5/6} (1+\tan[x]^2) + 9 \tan[x]^4 \left( 4 + \sqrt{-2-3 \tan[x]^2} \right) + \tan[x]^2 \right. \right. \\
& \quad \left. \left. \left( 60 - 7 (-2-3 \tan[x]^2)^{1/3} + 15 \sqrt{-2-3 \tan[x]^2} \right) + 2 \left( 12 - 2 (-2-3 \tan[x]^2)^{1/3} + 3 \sqrt{-2-3 \tan[x]^2} + (-2-3 \tan[x]^2)^{5/6} \right) \right) \right) \right) / \\
& \left( 10 \times 2^{1/6} \left( -1 + \sqrt{\frac{-5 + \cos[2x]}{1 + \cos[2x]}} \right) (1 - 3 \operatorname{Sec}[x]^2)^{5/6} \left( 6 + (1 - 3 \operatorname{Sec}[x]^2)^{1/3} + \cos[2x] (1 - 3 \operatorname{Sec}[x]^2)^{1/3} \right) \right. \\
& \quad (-4 - 6 \tan[x]^2)^{5/6} \\
& \quad \left( -8 \operatorname{AppellF1} \left[ 1, \frac{1}{2}, 1, 2, -\frac{3}{2} \tan[x]^2, -\tan[x]^2 \right] + \right. \\
& \quad \left( 4 \operatorname{AppellF1} \left[ 2, \frac{1}{2}, 2, 3, -\frac{3}{2} \tan[x]^2, -\tan[x]^2 \right] + 3 \operatorname{AppellF1} \left[ 2, \frac{3}{2}, 1, 3, -\frac{3}{2} \tan[x]^2, -\tan[x]^2 \right] \right) \tan[x]^2 \right) \\
& \quad \left( 1152 \operatorname{AppellF1} \left[ 1, \frac{1}{2}, 1, 2, -\frac{3}{2} \tan[x]^2, -\tan[x]^2 \right]^2 \tan[x]^3 + 2880 \operatorname{AppellF1} \left[ 1, \frac{1}{2}, 1, 2, -\frac{3}{2} \tan[x]^2, -\tan[x]^2 \right]^2 \tan[x]^5 - \right. \\
& \quad 1152 \operatorname{AppellF1} \left[ 1, \frac{1}{2}, 1, 2, -\frac{3}{2} \tan[x]^2, -\tan[x]^2 \right] \operatorname{AppellF1} \left[ 2, \frac{1}{2}, 2, 3, -\frac{3}{2} \tan[x]^2, -\tan[x]^2 \right] \tan[x]^5 - \\
& \quad 864 \operatorname{AppellF1} \left[ 1, \frac{1}{2}, 1, 2, -\frac{3}{2} \tan[x]^2, -\tan[x]^2 \right] \operatorname{AppellF1} \left[ 2, \frac{3}{2}, 1, 3, -\frac{3}{2} \tan[x]^2, -\tan[x]^2 \right] \tan[x]^5 + \\
& \quad 1728 \operatorname{AppellF1} \left[ 1, \frac{1}{2}, 1, 2, -\frac{3}{2} \tan[x]^2, -\tan[x]^2 \right]^2 \tan[x]^7 - 2880 \operatorname{AppellF1} \left[ 1, \frac{1}{2}, 1, 2, -\frac{3}{2} \tan[x]^2, -\tan[x]^2 \right] \\
& \quad \operatorname{AppellF1} \left[ 2, \frac{1}{2}, 2, 3, -\frac{3}{2} \tan[x]^2, -\tan[x]^2 \right] \tan[x]^7 + 288 \operatorname{AppellF1} \left[ 2, \frac{1}{2}, 2, 3, -\frac{3}{2} \tan[x]^2, -\tan[x]^2 \right]^2 \tan[x]^7 - \\
& \quad \left. 2160 \operatorname{AppellF1} \left[ 1, \frac{1}{2}, 1, 2, -\frac{3}{2} \tan[x]^2, -\tan[x]^2 \right] \operatorname{AppellF1} \left[ 2, \frac{3}{2}, 1, 3, -\frac{3}{2} \tan[x]^2, -\tan[x]^2 \right] \tan[x]^7 + \right.
\end{aligned}$$



$$\begin{aligned}
& 432 \operatorname{AppellF1}\left[2, \frac{1}{2}, 2, 3, -\frac{3}{2} \tan[x]^2, -\tan[x]^2\right] \operatorname{AppellF1}\left[2, \frac{3}{2}, 1, 3, -\frac{3}{2} \tan[x]^2, -\tan[x]^2\right] \tan[x]^7 + \\
& 162 \operatorname{AppellF1}\left[2, \frac{3}{2}, 1, 3, -\frac{3}{2} \tan[x]^2, -\tan[x]^2\right]^2 \tan[x]^7 - 1728 \operatorname{AppellF1}\left[1, \frac{1}{2}, 1, 2, -\frac{3}{2} \tan[x]^2, -\tan[x]^2\right] \\
& \operatorname{AppellF1}\left[2, \frac{1}{2}, 2, 3, -\frac{3}{2} \tan[x]^2, -\tan[x]^2\right] \tan[x]^9 + 720 \operatorname{AppellF1}\left[2, \frac{1}{2}, 2, 3, -\frac{3}{2} \tan[x]^2, -\tan[x]^2\right]^2 \tan[x]^9 - \\
& 1296 \operatorname{AppellF1}\left[1, \frac{1}{2}, 1, 2, -\frac{3}{2} \tan[x]^2, -\tan[x]^2\right] \operatorname{AppellF1}\left[2, \frac{3}{2}, 1, 3, -\frac{3}{2} \tan[x]^2, -\tan[x]^2\right] \tan[x]^9 + \\
& 1080 \operatorname{AppellF1}\left[2, \frac{1}{2}, 2, 3, -\frac{3}{2} \tan[x]^2, -\tan[x]^2\right] \operatorname{AppellF1}\left[2, \frac{3}{2}, 1, 3, -\frac{3}{2} \tan[x]^2, -\tan[x]^2\right] \tan[x]^9 + \\
& 405 \operatorname{AppellF1}\left[2, \frac{3}{2}, 1, 3, -\frac{3}{2} \tan[x]^2, -\tan[x]^2\right]^2 \tan[x]^9 + 432 \operatorname{AppellF1}\left[2, \frac{1}{2}, 2, 3, -\frac{3}{2} \tan[x]^2, -\tan[x]^2\right]^2 \tan[x]^{11} + \\
& 648 \operatorname{AppellF1}\left[2, \frac{1}{2}, 2, 3, -\frac{3}{2} \tan[x]^2, -\tan[x]^2\right] \operatorname{AppellF1}\left[2, \frac{3}{2}, 1, 3, -\frac{3}{2} \tan[x]^2, -\tan[x]^2\right] \tan[x]^{11} + \\
& 243 \operatorname{AppellF1}\left[2, \frac{3}{2}, 1, 3, -\frac{3}{2} \tan[x]^2, -\tan[x]^2\right]^2 \tan[x]^{11} + \\
& 720 \operatorname{AppellF1}\left[1, \frac{1}{2}, 1, 2, -\frac{3}{2} \tan[x]^2, -\tan[x]^2\right]^2 \tan[x]^3 (-2 - 3 \tan[x]^2)^{1/3} - \\
& 192 \operatorname{AppellF1}\left[1, \frac{1}{2}, 1, 2, -\frac{3}{2} \tan[x]^2, -\tan[x]^2\right] \operatorname{AppellF1}\left[2, \frac{1}{2}, 2, 3, -\frac{3}{2} \tan[x]^2, -\tan[x]^2\right] \tan[x]^3 (-2 - 3 \tan[x]^2)^{1/3} - \\
& 144 \operatorname{AppellF1}\left[1, \frac{1}{2}, 1, 2, -\frac{3}{2} \tan[x]^2, -\tan[x]^2\right] \operatorname{AppellF1}\left[2, \frac{3}{2}, 1, 3, -\frac{3}{2} \tan[x]^2, -\tan[x]^2\right] \tan[x]^3 (-2 - 3 \tan[x]^2)^{1/3} + \\
& 1008 \operatorname{AppellF1}\left[1, \frac{1}{2}, 1, 2, -\frac{3}{2} \tan[x]^2, -\tan[x]^2\right]^2 \tan[x]^5 (-2 - 3 \tan[x]^2)^{1/3} - \\
& 1032 \operatorname{AppellF1}\left[1, \frac{1}{2}, 1, 2, -\frac{3}{2} \tan[x]^2, -\tan[x]^2\right] \operatorname{AppellF1}\left[2, \frac{1}{2}, 2, 3, -\frac{3}{2} \tan[x]^2, -\tan[x]^2\right] \tan[x]^5 (-2 - 3 \tan[x]^2)^{1/3} - \\
& 774 \operatorname{AppellF1}\left[1, \frac{1}{2}, 1, 2, -\frac{3}{2} \tan[x]^2, -\tan[x]^2\right] \operatorname{AppellF1}\left[2, \frac{3}{2}, 1, 3, -\frac{3}{2} \tan[x]^2, -\tan[x]^2\right] \tan[x]^5 (-2 - 3 \tan[x]^2)^{1/3} + \\
& 128 \operatorname{AppellF1}\left[1, \frac{1}{2}, 1, 2, -\frac{3}{2} \tan[x]^2, -\tan[x]^2\right] \operatorname{AppellF1}\left[3, \frac{1}{2}, 3, 4, -\frac{3}{2} \tan[x]^2, -\tan[x]^2\right] \tan[x]^5 (-2 - 3 \tan[x]^2)^{1/3} + \\
& 96 \operatorname{AppellF1}\left[1, \frac{1}{2}, 1, 2, -\frac{3}{2} \tan[x]^2, -\tan[x]^2\right] \operatorname{AppellF1}\left[3, \frac{3}{2}, 2, 4, -\frac{3}{2} \tan[x]^2, -\tan[x]^2\right] \tan[x]^5 (-2 - 3 \tan[x]^2)^{1/3} + \\
& 108 \operatorname{AppellF1}\left[1, \frac{1}{2}, 1, 2, -\frac{3}{2} \tan[x]^2, -\tan[x]^2\right] \operatorname{AppellF1}\left[3, \frac{5}{2}, 1, 4, -\frac{3}{2} \tan[x]^2, -\tan[x]^2\right] \tan[x]^5 (-2 - 3 \tan[x]^2)^{1/3} - \\
& 1080 \operatorname{AppellF1}\left[1, \frac{1}{2}, 1, 2, -\frac{3}{2} \tan[x]^2, -\tan[x]^2\right] \operatorname{AppellF1}\left[2, \frac{1}{2}, 2, 3, -\frac{3}{2} \tan[x]^2, -\tan[x]^2\right] \tan[x]^7 (-2 - 3 \tan[x]^2)^{1/3} + \\
& 96 \operatorname{AppellF1}\left[2, \frac{1}{2}, 2, 3, -\frac{3}{2} \tan[x]^2, -\tan[x]^2\right]^2 \tan[x]^7 (-2 - 3 \tan[x]^2)^{1/3} - \\
& 810 \operatorname{AppellF1}\left[1, \frac{1}{2}, 1, 2, -\frac{3}{2} \tan[x]^2, -\tan[x]^2\right] \operatorname{AppellF1}\left[2, \frac{3}{2}, 1, 3, -\frac{3}{2} \tan[x]^2, -\tan[x]^2\right] \tan[x]^7 (-2 - 3 \tan[x]^2)^{1/3} +
\end{aligned}$$

$$\begin{aligned}
& 144 \operatorname{AppellF1}\left[2, \frac{1}{2}, 2, 3, -\frac{3}{2} \tan[x]^2, -\tan[x]^2\right] \operatorname{AppellF1}\left[2, \frac{3}{2}, 1, 3, -\frac{3}{2} \tan[x]^2, -\tan[x]^2\right] \tan[x]^7 (-2 - 3 \tan[x]^2)^{1/3} + \\
& 54 \operatorname{AppellF1}\left[2, \frac{3}{2}, 1, 3, -\frac{3}{2} \tan[x]^2, -\tan[x]^2\right]^2 \tan[x]^7 (-2 - 3 \tan[x]^2)^{1/3} + \\
& 320 \operatorname{AppellF1}\left[1, \frac{1}{2}, 1, 2, -\frac{3}{2} \tan[x]^2, -\tan[x]^2\right] \operatorname{AppellF1}\left[3, \frac{1}{2}, 3, 4, -\frac{3}{2} \tan[x]^2, -\tan[x]^2\right] \tan[x]^7 (-2 - 3 \tan[x]^2)^{1/3} + \\
& 240 \operatorname{AppellF1}\left[1, \frac{1}{2}, 1, 2, -\frac{3}{2} \tan[x]^2, -\tan[x]^2\right] \operatorname{AppellF1}\left[3, \frac{3}{2}, 2, 4, -\frac{3}{2} \tan[x]^2, -\tan[x]^2\right] \tan[x]^7 (-2 - 3 \tan[x]^2)^{1/3} + \\
& 270 \operatorname{AppellF1}\left[1, \frac{1}{2}, 1, 2, -\frac{3}{2} \tan[x]^2, -\tan[x]^2\right] \operatorname{AppellF1}\left[3, \frac{5}{2}, 1, 4, -\frac{3}{2} \tan[x]^2, -\tan[x]^2\right] \tan[x]^7 (-2 - 3 \tan[x]^2)^{1/3} + \\
& 144 \operatorname{AppellF1}\left[2, \frac{1}{2}, 2, 3, -\frac{3}{2} \tan[x]^2, -\tan[x]^2\right]^2 \tan[x]^9 (-2 - 3 \tan[x]^2)^{1/3} + \\
& 216 \operatorname{AppellF1}\left[2, \frac{1}{2}, 2, 3, -\frac{3}{2} \tan[x]^2, -\tan[x]^2\right] \operatorname{AppellF1}\left[2, \frac{3}{2}, 1, 3, -\frac{3}{2} \tan[x]^2, -\tan[x]^2\right] \tan[x]^9 (-2 - 3 \tan[x]^2)^{1/3} + \\
& 81 \operatorname{AppellF1}\left[2, \frac{3}{2}, 1, 3, -\frac{3}{2} \tan[x]^2, -\tan[x]^2\right]^2 \tan[x]^9 (-2 - 3 \tan[x]^2)^{1/3} + \\
& 192 \operatorname{AppellF1}\left[1, \frac{1}{2}, 1, 2, -\frac{3}{2} \tan[x]^2, -\tan[x]^2\right] \operatorname{AppellF1}\left[3, \frac{1}{2}, 3, 4, -\frac{3}{2} \tan[x]^2, -\tan[x]^2\right] \tan[x]^9 (-2 - 3 \tan[x]^2)^{1/3} + \\
& 144 \operatorname{AppellF1}\left[1, \frac{1}{2}, 1, 2, -\frac{3}{2} \tan[x]^2, -\tan[x]^2\right] \operatorname{AppellF1}\left[3, \frac{3}{2}, 2, 4, -\frac{3}{2} \tan[x]^2, -\tan[x]^2\right] \tan[x]^9 (-2 - 3 \tan[x]^2)^{1/3} + \\
& 162 \operatorname{AppellF1}\left[1, \frac{1}{2}, 1, 2, -\frac{3}{2} \tan[x]^2, -\tan[x]^2\right] \operatorname{AppellF1}\left[3, \frac{5}{2}, 1, 4, -\frac{3}{2} \tan[x]^2, -\tan[x]^2\right] \tan[x]^9 (-2 - 3 \tan[x]^2)^{1/3} + \\
& 1152 \operatorname{AppellF1}\left[1, \frac{1}{2}, 1, 2, -\frac{3}{2} \tan[x]^2, -\tan[x]^2\right]^2 \tan[x]^3 \sqrt{-2 - 3 \tan[x]^2} + \\
& 2880 \operatorname{AppellF1}\left[1, \frac{1}{2}, 1, 2, -\frac{3}{2} \tan[x]^2, -\tan[x]^2\right]^2 \tan[x]^5 \sqrt{-2 - 3 \tan[x]^2} - \\
& 1152 \operatorname{AppellF1}\left[1, \frac{1}{2}, 1, 2, -\frac{3}{2} \tan[x]^2, -\tan[x]^2\right] \operatorname{AppellF1}\left[2, \frac{1}{2}, 2, 3, -\frac{3}{2} \tan[x]^2, -\tan[x]^2\right] \tan[x]^5 \sqrt{-2 - 3 \tan[x]^2} - \\
& 864 \operatorname{AppellF1}\left[1, \frac{1}{2}, 1, 2, -\frac{3}{2} \tan[x]^2, -\tan[x]^2\right] \operatorname{AppellF1}\left[2, \frac{3}{2}, 1, 3, -\frac{3}{2} \tan[x]^2, -\tan[x]^2\right] \tan[x]^5 \sqrt{-2 - 3 \tan[x]^2} + \\
& 1728 \operatorname{AppellF1}\left[1, \frac{1}{2}, 1, 2, -\frac{3}{2} \tan[x]^2, -\tan[x]^2\right]^2 \tan[x]^7 \sqrt{-2 - 3 \tan[x]^2} - \\
& 2880 \operatorname{AppellF1}\left[1, \frac{1}{2}, 1, 2, -\frac{3}{2} \tan[x]^2, -\tan[x]^2\right] \operatorname{AppellF1}\left[2, \frac{1}{2}, 2, 3, -\frac{3}{2} \tan[x]^2, -\tan[x]^2\right] \tan[x]^7 \sqrt{-2 - 3 \tan[x]^2} + \\
& 288 \operatorname{AppellF1}\left[2, \frac{1}{2}, 2, 3, -\frac{3}{2} \tan[x]^2, -\tan[x]^2\right]^2 \tan[x]^7 \sqrt{-2 - 3 \tan[x]^2} - \\
& 2160 \operatorname{AppellF1}\left[1, \frac{1}{2}, 1, 2, -\frac{3}{2} \tan[x]^2, -\tan[x]^2\right] \operatorname{AppellF1}\left[2, \frac{3}{2}, 1, 3, -\frac{3}{2} \tan[x]^2, -\tan[x]^2\right] \tan[x]^7 \sqrt{-2 - 3 \tan[x]^2} + \\
& 432 \operatorname{AppellF1}\left[2, \frac{1}{2}, 2, 3, -\frac{3}{2} \tan[x]^2, -\tan[x]^2\right] \operatorname{AppellF1}\left[2, \frac{3}{2}, 1, 3, -\frac{3}{2} \tan[x]^2, -\tan[x]^2\right] \tan[x]^7 \sqrt{-2 - 3 \tan[x]^2} +
\end{aligned}$$

$$\begin{aligned}
& 162 \operatorname{AppellF1}\left[2, \frac{3}{2}, 1, 3, -\frac{3}{2} \tan[x]^2, -\tan[x]^2\right]^2 \tan[x]^7 \sqrt{-2-3 \tan[x]^2} - \\
& 1728 \operatorname{AppellF1}\left[1, \frac{1}{2}, 1, 2, -\frac{3}{2} \tan[x]^2, -\tan[x]^2\right] \operatorname{AppellF1}\left[2, \frac{1}{2}, 2, 3, -\frac{3}{2} \tan[x]^2, -\tan[x]^2\right] \tan[x]^9 \sqrt{-2-3 \tan[x]^2} + \\
& 720 \operatorname{AppellF1}\left[2, \frac{1}{2}, 2, 3, -\frac{3}{2} \tan[x]^2, -\tan[x]^2\right]^2 \tan[x]^9 \sqrt{-2-3 \tan[x]^2} - \\
& 1296 \operatorname{AppellF1}\left[1, \frac{1}{2}, 1, 2, -\frac{3}{2} \tan[x]^2, -\tan[x]^2\right] \operatorname{AppellF1}\left[2, \frac{3}{2}, 1, 3, -\frac{3}{2} \tan[x]^2, -\tan[x]^2\right] \tan[x]^9 \sqrt{-2-3 \tan[x]^2} + \\
& 1080 \operatorname{AppellF1}\left[2, \frac{1}{2}, 2, 3, -\frac{3}{2} \tan[x]^2, -\tan[x]^2\right] \operatorname{AppellF1}\left[2, \frac{3}{2}, 1, 3, -\frac{3}{2} \tan[x]^2, -\tan[x]^2\right] \tan[x]^9 \sqrt{-2-3 \tan[x]^2} + \\
& 405 \operatorname{AppellF1}\left[2, \frac{3}{2}, 1, 3, -\frac{3}{2} \tan[x]^2, -\tan[x]^2\right]^2 \tan[x]^9 \sqrt{-2-3 \tan[x]^2} + \\
& 432 \operatorname{AppellF1}\left[2, \frac{1}{2}, 2, 3, -\frac{3}{2} \tan[x]^2, -\tan[x]^2\right]^2 \tan[x]^{11} \sqrt{-2-3 \tan[x]^2} + \\
& 648 \operatorname{AppellF1}\left[2, \frac{1}{2}, 2, 3, -\frac{3}{2} \tan[x]^2, -\tan[x]^2\right] \operatorname{AppellF1}\left[2, \frac{3}{2}, 1, 3, -\frac{3}{2} \tan[x]^2, -\tan[x]^2\right] \tan[x]^{11} \sqrt{-2-3 \tan[x]^2} + \\
& 243 \operatorname{AppellF1}\left[2, \frac{3}{2}, 1, 3, -\frac{3}{2} \tan[x]^2, -\tan[x]^2\right]^2 \tan[x]^{11} \sqrt{-2-3 \tan[x]^2} + 384 \operatorname{AppellF1}\left[1, \frac{1}{2}, 1, 2, -\frac{3}{2} \tan[x]^2, -\tan[x]^2\right]^2 \\
& \quad \tan[x]^3 (-2-3 \tan[x]^2)^{5/6} + 576 \operatorname{AppellF1}\left[1, \frac{1}{2}, 1, 2, -\frac{3}{2} \tan[x]^2, -\tan[x]^2\right]^2 \tan[x]^5 (-2-3 \tan[x]^2)^{5/6} - \\
& 384 \operatorname{AppellF1}\left[1, \frac{1}{2}, 1, 2, -\frac{3}{2} \tan[x]^2, -\tan[x]^2\right] \operatorname{AppellF1}\left[2, \frac{1}{2}, 2, 3, -\frac{3}{2} \tan[x]^2, -\tan[x]^2\right] \tan[x]^5 (-2-3 \tan[x]^2)^{5/6} - \\
& 288 \operatorname{AppellF1}\left[1, \frac{1}{2}, 1, 2, -\frac{3}{2} \tan[x]^2, -\tan[x]^2\right] \operatorname{AppellF1}\left[2, \frac{3}{2}, 1, 3, -\frac{3}{2} \tan[x]^2, -\tan[x]^2\right] \tan[x]^5 (-2-3 \tan[x]^2)^{5/6} - \\
& 576 \operatorname{AppellF1}\left[1, \frac{1}{2}, 1, 2, -\frac{3}{2} \tan[x]^2, -\tan[x]^2\right] \operatorname{AppellF1}\left[2, \frac{1}{2}, 2, 3, -\frac{3}{2} \tan[x]^2, -\tan[x]^2\right] \tan[x]^7 (-2-3 \tan[x]^2)^{5/6} + \\
& 96 \operatorname{AppellF1}\left[2, \frac{1}{2}, 2, 3, -\frac{3}{2} \tan[x]^2, -\tan[x]^2\right]^2 \tan[x]^7 (-2-3 \tan[x]^2)^{5/6} - \\
& 432 \operatorname{AppellF1}\left[1, \frac{1}{2}, 1, 2, -\frac{3}{2} \tan[x]^2, -\tan[x]^2\right] \operatorname{AppellF1}\left[2, \frac{3}{2}, 1, 3, -\frac{3}{2} \tan[x]^2, -\tan[x]^2\right] \tan[x]^7 (-2-3 \tan[x]^2)^{5/6} + \\
& 144 \operatorname{AppellF1}\left[2, \frac{1}{2}, 2, 3, -\frac{3}{2} \tan[x]^2, -\tan[x]^2\right] \operatorname{AppellF1}\left[2, \frac{3}{2}, 1, 3, -\frac{3}{2} \tan[x]^2, -\tan[x]^2\right] \tan[x]^7 (-2-3 \tan[x]^2)^{5/6} + \\
& 54 \operatorname{AppellF1}\left[2, \frac{3}{2}, 1, 3, -\frac{3}{2} \tan[x]^2, -\tan[x]^2\right]^2 \tan[x]^7 (-2-3 \tan[x]^2)^{5/6} + \\
& 144 \operatorname{AppellF1}\left[2, \frac{1}{2}, 2, 3, -\frac{3}{2} \tan[x]^2, -\tan[x]^2\right]^2 \tan[x]^9 (-2-3 \tan[x]^2)^{5/6} + \\
& 216 \operatorname{AppellF1}\left[2, \frac{1}{2}, 2, 3, -\frac{3}{2} \tan[x]^2, -\tan[x]^2\right] \operatorname{AppellF1}\left[2, \frac{3}{2}, 1, 3, -\frac{3}{2} \tan[x]^2, -\tan[x]^2\right] \tan[x]^9 (-2-3 \tan[x]^2)^{5/6} +
\end{aligned}$$

$$81 \operatorname{AppellF1}\left[2, \frac{3}{2}, 1, 3, -\frac{3}{2} \tan[x]^2, -\tan[x]^2\right]^2 \tan[x]^9 (-2 - 3 \tan[x]^2)^{5/6} \Bigg)$$

**Problem 447: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{\sec[x]^2 (-\cos[2x] + 2 \tan[x]^2)}{(\tan[x] \tan[2x])^{3/2}} dx$$

Optimal (type 3, 100 leaves, ? steps):

$$2 \operatorname{ArcTanh}\left[\frac{\tan[x]}{\sqrt{\tan[x] \tan[2x]}}\right] - \frac{11 \operatorname{ArcTanh}\left[\frac{\sqrt{2} \tan[x]}{\sqrt{\tan[x] \tan[2x]}}\right]}{4 \sqrt{2}} + \frac{\tan[x]}{2 (\tan[x] \tan[2x])^{3/2}} + \frac{2 \tan[x]^3}{3 (\tan[x] \tan[2x])^{3/2}} + \frac{3 \tan[x]}{4 \sqrt{\tan[x] \tan[2x]}}$$

Result (type 6, 207 leaves):

$$\left( (-\cos[2x] + 2 \tan[x]^2) \left( -3 \cot[x] - 4 \cos[x] \sin[x] + 18 \sin[x]^2 \tan[x] - 4 \tan[x]^3 - 9 \operatorname{ArcTan}\left[\sqrt{-1 + \tan[x]^2}\right] \cos[x] \sin[x] \sqrt{-1 + \tan[x]^2} - \right. \right. \\ \left. \left. \left( 72 \operatorname{AppellF1}\left[\frac{1}{2}, -\frac{1}{2}, 1, \frac{3}{2}, \cot[x]^2, -\cot[x]^2\right] \cos[2x] \sin[x]^2 \tan[x] \right) \right) \right) / \left( 2 \operatorname{AppellF1}\left[\frac{3}{2}, -\frac{1}{2}, 2, \frac{5}{2}, \cot[x]^2, -\cot[x]^2\right] + \right. \\ \left. \operatorname{AppellF1}\left[\frac{3}{2}, \frac{1}{2}, 1, \frac{5}{2}, \cot[x]^2, -\cot[x]^2\right] - 3 \operatorname{AppellF1}\left[\frac{1}{2}, -\frac{1}{2}, 1, \frac{3}{2}, \cot[x]^2, -\cot[x]^2\right] \tan[x]^2 \right) \\ \left. \tan[2x]^2 \right) / \left( 6 (-3 + 6 \cos[2x] + \cos[4x]) (\tan[x] \tan[2x])^{3/2} \right)$$

**Problem 448: Result unnecessarily involves higher level functions.**

$$\int \frac{\tan[x]}{(a^3 - b^3 \cos[x]^n)^{4/3}} dx$$

Optimal (type 3, 112 leaves, 7 steps):

$$-\frac{\sqrt{3} \operatorname{ArcTan}\left[\frac{a+2(a^3-b^3 \cos[x]^n)^{1/3}}{\sqrt{3} a}\right]}{a^4 n} - \frac{3}{a^3 n (a^3 - b^3 \cos[x]^n)^{1/3}} + \frac{\log[\cos[x]]}{2 a^4} - \frac{3 \log[a - (a^3 - b^3 \cos[x]^n)^{1/3}]}{2 a^4 n}$$

Result (type 5, 71 leaves):

$$\frac{3 \left( -1 + \left( 1 - \frac{a^3 \cos[x]^n}{b^3} \right)^{1/3} \operatorname{Hypergeometric2F1}\left[\frac{1}{3}, \frac{1}{3}, \frac{4}{3}, \frac{a^3 \cos[x]^n}{b^3}\right] \right)}{a^3 n (a^3 - b^3 \cos[x]^n)^{1/3}}$$

**Problem 449: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int (1 + 2 \cos [x]^9)^{5/6} \tan [x] \, dx$$

Optimal (type 3, 95 leaves, 14 steps):

$$\frac{\text{ArcTan} \left[ \frac{1 - (1 + 2 \cos [x]^9)^{1/3}}{\sqrt{3} (1 + 2 \cos [x]^9)^{1/6}} \right]}{3 \sqrt{3}} + \frac{1}{3} \text{ArcTanh} \left[ (1 + 2 \cos [x]^9)^{1/6} \right] - \frac{1}{9} \text{ArcTanh} \left[ \sqrt{1 + 2 \cos [x]^9} \right] - \frac{2}{15} (1 + 2 \cos [x]^9)^{5/6}$$

Result (type 5, 579 leaves):

$$\left( (128 + 126 \cos [x] + 84 \cos [3x] + 36 \cos [5x] + 9 \cos [7x] + \cos [9x])^{5/6} \right. \\ \left. (1 + \cot [x]^2)^5 \sin [x]^2 \left( 1 + 5 \cot [x]^2 + 10 \cot [x]^4 + 10 \cot [x]^6 + 5 \cot [x]^8 + \cot [x]^{10} + 2 \cot [x]^{10} \sqrt{1 + \tan [x]^2} \right) \right. \\ \left. \left( \frac{1 + 5 \cot [x]^2 + 10 \cot [x]^4 + 10 \cot [x]^6 + 5 \cot [x]^8 + \cot [x]^{10} + 2 \cot [x]^{10} \sqrt{1 + \tan [x]^2}}{(1 + \cot [x]^2)^5} \right)^{1/6} \right. \\ \left. \left( -2 \left( 1 + 5 \tan [x]^2 + 10 \tan [x]^4 + 10 \tan [x]^6 + 5 \tan [x]^8 + \tan [x]^{10} + 2 \sqrt{1 + \tan [x]^2} \right) + \right. \right. \\ \left. \left. 5 \times 2^{5/6} \text{Hypergeometric2F1} \left[ \frac{1}{6}, \frac{1}{6}, \frac{7}{6}, -\frac{1}{2} (1 + \tan [x]^2)^{9/2} \right] (1 + \tan [x]^2)^5 \right. \right. \\ \left. \left. \left( 2 + \sqrt{1 + \tan [x]^2} + 4 \tan [x]^2 \sqrt{1 + \tan [x]^2} + 6 \tan [x]^4 \sqrt{1 + \tan [x]^2} + 4 \tan [x]^6 \sqrt{1 + \tan [x]^2} + \tan [x]^8 \sqrt{1 + \tan [x]^2} \right)^{1/6} \right) \right) / \\ \left( 480 \times 2^{5/6} (1 + \tan [x]^2)^{9/2} \left( \frac{1 + 5 \tan [x]^2 + 10 \tan [x]^4 + 10 \tan [x]^6 + 5 \tan [x]^8 + \tan [x]^{10} + 2 \sqrt{1 + \tan [x]^2}}{(1 + \tan [x]^2)^5} \right)^{1/6} \right. \\ \left. \left( 4 \cot [x]^8 + 20 \cot [x]^{10} + 40 \cot [x]^{12} + 40 \cot [x]^{14} + 20 \cot [x]^{16} + 4 \cot [x]^{18} + \sqrt{1 + \tan [x]^2} + 9 \cot [x]^2 \sqrt{1 + \tan [x]^2} + \right. \right. \\ \left. \left. 36 \cot [x]^4 \sqrt{1 + \tan [x]^2} + 84 \cot [x]^6 \sqrt{1 + \tan [x]^2} + 126 \cot [x]^8 \sqrt{1 + \tan [x]^2} + 126 \cot [x]^{10} \sqrt{1 + \tan [x]^2} + \right. \right. \\ \left. \left. 84 \cot [x]^{12} \sqrt{1 + \tan [x]^2} + 36 \cot [x]^{14} \sqrt{1 + \tan [x]^2} + 9 \cot [x]^{16} \sqrt{1 + \tan [x]^2} + 5 \cot [x]^{18} \sqrt{1 + \tan [x]^2} \right) \right) \right)$$

### Problem 451: Result more than twice size of optimal antiderivative.

$$\int \frac{\sec [x]^2 \tan [x] \left(1 + \left(1 - 8 \tan [x]^2\right)^{1/3}\right)}{\left(1 - 8 \tan [x]^2\right)^{2/3}} dx$$

Optimal (type 3, 20 leaves, 2 steps):

$$-\frac{3}{32} \left(1 + \left(1 - 8 \tan [x]^2\right)^{1/3}\right)^2$$

Result (type 3, 42 leaves):

$$-\frac{3 \left(-7 + 9 \cos [2 x]\right) \sec [x]^2 \left(2 + \left(1 - 8 \tan [x]^2\right)^{1/3}\right)}{64 \left(1 - 8 \tan [x]^2\right)^{2/3}}$$

### Problem 452: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{\csc [x] \sec [x] \left(1 + \left(1 - 8 \tan [x]^2\right)^{1/3}\right)}{\left(1 - 8 \tan [x]^2\right)^{2/3}} dx$$

Optimal (type 3, 27 leaves, 15 steps):

$$-\log [\tan [x]] + \frac{3}{2} \log \left[1 - \left(1 - 8 \tan [x]^2\right)^{1/3}\right]$$

Result (type 5, 93 leaves):

$$-\frac{3 \left(8 - \cot [x]^2\right)^{2/3} \text{Hypergeometric2F1}\left[\frac{2}{3}, \frac{2}{3}, \frac{5}{3}, \frac{\cot [x]^2}{8}\right]}{16 \left(1 - 8 \tan [x]^2\right)^{2/3}} - \frac{3 \left(8 - \cot [x]^2\right)^{1/3} \text{Hypergeometric2F1}\left[\frac{1}{3}, \frac{1}{3}, \frac{4}{3}, \frac{\cot [x]^2}{8}\right]}{4 \left(1 - 8 \tan [x]^2\right)^{1/3}}$$

### Problem 453: Result unnecessarily involves higher level functions.

$$\int \frac{\left(5 \cos [x]^2 - \sqrt{-1 + 5 \sin [x]^2}\right) \tan [x]}{\left(-1 + 5 \sin [x]^2\right)^{1/4} \left(2 + \sqrt{-1 + 5 \sin [x]^2}\right)} dx$$

Optimal (type 3, 101 leaves, 14 steps):

$$-\frac{3 \operatorname{ArcTan}\left[\frac{(-1+5 \sin [x]^2)^{1/4}}{\sqrt{2}}\right]}{\sqrt{2}} - \frac{\operatorname{ArcTanh}\left[\frac{(-1+5 \sin [x]^2)^{1/4}}{\sqrt{2}}\right]}{2 \sqrt{2}} + 2(-1+5 \sin [x]^2)^{1/4} - \frac{(-1+5 \sin [x]^2)^{1/4}}{2\left(2+\sqrt{-1+5 \sin [x]^2}\right)}$$

Result (type 5, 158 leaves):

$$-\frac{1}{60(3-5 \cos [2 x])^{3/4}}\left(3 \times 2^{1/4}(-3+5 \cos [2 x])\left(8 \sqrt{2}+\sqrt{3-5 \cos [2 x]}+10 \sqrt{2} \cos [2 x]\right) \sec [x]^2 -\right. \\ \left.30 \times 5^{3/4} \sqrt{3-5 \cos [2 x]} \operatorname{Hypergeometric2F1}\left[\frac{1}{4}, \frac{1}{4}, \frac{5}{4}, \frac{4 \sec [x]^2}{5}\right]\left((-3+5 \cos [2 x]) \sec [x]^2\right)^{1/4} +\right. \\ \left.28 \times 5^{1/4} \operatorname{Hypergeometric2F1}\left[\frac{3}{4}, \frac{3}{4}, \frac{7}{4}, \frac{4 \sec [x]^2}{5}\right]\left(2-8 \tan [x]^2\right)^{3/4}\right)$$

**Problem 454: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \cos [x]^3 \cos [2 x]^{2/3} \sin [x] dx$$

Optimal (type 3, 25 leaves, 4 steps):

$$-\frac{3}{40} \cos [2 x]^{5/3} - \frac{3}{64} \cos [2 x]^{8/3}$$

Result (type 5, 140 leaves):

$$-\frac{3}{40} \cos [2 x]^{5/3} - \\ \left(3 e^{-6 i x}\left(1+e^{4 i x}\right)^{1/3}\left(\left(1+e^{4 i x}\right)^{2/3}\left(1+e^{8 i x}\right)+2 e^{4 i x} \operatorname{Hypergeometric2F1}\left[-\frac{1}{3}, \frac{1}{3}, \frac{2}{3}, -e^{4 i x}\right]+e^{8 i x} \operatorname{Hypergeometric2F1}\left[\frac{1}{3}, \frac{2}{3}, \frac{5}{3}, -e^{4 i x}\right]\right)\right) / \\ \left(256 \times 2^{2/3}\left(e^{-2 i x}+e^{2 i x}\right)^{1/3}\right)$$

**Problem 455: Result unnecessarily involves higher level functions.**

$$\int \frac{\sin [x]^6 \tan [x]}{\cos [2 x]^{3/4}} dx$$

Optimal (type 3, 102 leaves, ? steps):

$$\frac{\operatorname{ArcTan}\left[\frac{1-\sqrt{\cos [2 x]}}{\sqrt{2} \cos [2 x]^{1/4}}\right]}{\sqrt{2}} - \frac{\operatorname{ArcTanh}\left[\frac{1+\sqrt{\cos [2 x]}}{\sqrt{2} \cos [2 x]^{1/4}}\right]}{\sqrt{2}} + \frac{7}{4} \cos [2 x]^{1/4} - \frac{1}{5} \cos [2 x]^{5/4} + \frac{1}{36} \cos [2 x]^{9/4}$$

Result (type 5, 59 leaves):

$$\frac{1}{360} \cos[2x]^{1/4} (635 - 72 \cos[2x] + 5 \cos[4x]) + \frac{2 \text{Hypergeometric2F1}\left[\frac{3}{4}, \frac{3}{4}, \frac{7}{4}, \frac{\sec[x]^2}{2}\right]}{3 (1 + \cos[2x])^{3/4}}$$

Problem 456: Result more than twice size of optimal antiderivative.

$$\int \sqrt{\tan[x] \tan[2x]} dx$$

Optimal (type 3, 17 leaves, 3 steps):

$$-\text{ArcTanh}\left[\frac{\tan[x]}{\sqrt{\tan[x] \tan[2x]}}\right]$$

Result (type 3, 45 leaves):

$$\frac{\text{ArcTanh}\left[\frac{\sqrt{2} \cos[x]}{\sqrt{\cos[2x]}}\right] \sqrt{\cos[2x]} \csc[x] \sqrt{\tan[x] \tan[2x]}}{\sqrt{2}}$$

Problem 488: Result more than twice size of optimal antiderivative.

$$\int x \sec[x] \tan[x]^3 dx$$

Optimal (type 3, 30 leaves, 5 steps):

$$\frac{5}{6} \text{ArcTanh}[\sin[x]] - x \sec[x] + \frac{1}{3} x \sec[x]^3 - \frac{1}{6} \sec[x] \tan[x]$$

Result (type 3, 104 leaves):

$$-\frac{1}{24} \sec[x]^3 \left(4x + 12x \cos[2x] + 5 \cos[3x] \log\left[\cos\left[\frac{x}{2}\right] - \sin\left[\frac{x}{2}\right]\right]\right) + 15 \cos[x] \left(\log\left[\cos\left[\frac{x}{2}\right] - \sin\left[\frac{x}{2}\right]\right] - \log\left[\cos\left[\frac{x}{2}\right] + \sin\left[\frac{x}{2}\right]\right]\right) - 5 \cos[3x] \log\left[\cos\left[\frac{x}{2}\right] + \sin\left[\frac{x}{2}\right]\right] + 2 \sin[2x]$$

Problem 506: Unable to integrate problem.

$$\int (a^{kx} + a^{lx})^n dx$$

Optimal (type 5, 72 leaves, 2 steps):



$$\frac{(1 + a^{(k-1)x}) (a^{kx} + a^{1x})^n \text{Hypergeometric2F1}\left[1, 1 + \frac{kn}{k-1}, 1 + \frac{1n}{k-1}, -a^{(k-1)x}\right]}{1 n \text{Log}[a]}$$

Result (type 8, 15 leaves):

$$\int (a^{kx} + a^{1x})^n dx$$

**Problem 511: Unable to integrate problem.**

$$\int (a^{kx} - a^{1x})^n dx$$

Optimal (type 5, 74 leaves, 2 steps):

$$\frac{(1 - a^{(k-1)x}) (a^{kx} - a^{1x})^n \text{Hypergeometric2F1}\left[1, 1 + \frac{kn}{k-1}, 1 + \frac{1n}{k-1}, a^{(k-1)x}\right]}{1 n \text{Log}[a]}$$

Result (type 8, 17 leaves):

$$\int (a^{kx} - a^{1x})^n dx$$

**Problem 523: Result is not expressed in closed-form.**

$$\int \frac{e^x}{b + a e^{3x}} dx$$

Optimal (type 3, 100 leaves, 7 steps):

$$-\frac{\text{ArcTan}\left[\frac{b^{1/3} - 2a^{1/3}e^x}{\sqrt{3}b^{1/3}}\right]}{\sqrt{3}a^{1/3}b^{2/3}} + \frac{\text{Log}[b^{1/3} + a^{1/3}e^x]}{2a^{1/3}b^{2/3}} - \frac{\text{Log}[b + a e^{3x}]}{6a^{1/3}b^{2/3}}$$

Result (type 7, 36 leaves):

$$\frac{\text{RootSum}\left[b + a \#1^3 \&, \frac{-x + \text{Log}[e^x - \#1]}{\#1^2} \&\right]}{3 a}$$

**Problem 528: Result unnecessarily involves higher level functions.**

$$\int (1 - 2e^{x/3})^{1/4} dx$$

Optimal (type 3, 54 leaves, 6 steps):

$$12 (1 - 2 e^{x/3})^{1/4} - 6 \operatorname{ArcTan}[(1 - 2 e^{x/3})^{1/4}] - 6 \operatorname{ArcTanh}[(1 - 2 e^{x/3})^{1/4}]$$

Result (type 5, 70 leaves):

$$- \frac{2 \left( -6 + 12 e^{x/3} + 2^{1/4} (2 - e^{-x/3})^{3/4} \operatorname{Hypergeometric2F1} \left[ \frac{3}{4}, \frac{3}{4}, \frac{7}{4}, \frac{e^{-x/3}}{2} \right] \right)}{(1 - 2 e^{x/3})^{3/4}}$$

**Problem 540: Unable to integrate problem.**

$$\int \frac{e^x (1 - x - x^2)}{\sqrt{1 - x^2}} dx$$

Optimal (type 3, 15 leaves, 1 step):

$$e^x \sqrt{1 - x^2}$$

Result (type 8, 27 leaves):

$$\int \frac{e^x (1 - x - x^2)}{\sqrt{1 - x^2}} dx$$

**Problem 552: Result more than twice size of optimal antiderivative.**

$$\int \frac{e^x}{1 + \operatorname{Cos}[x]} dx$$

Optimal (type 5, 28 leaves, 2 steps):

$$(1 - i) e^{(1+i)x} \operatorname{Hypergeometric2F1}[1 - i, 2, 2 - i, -e^{ix}]$$

Result (type 5, 89 leaves):

$$- \frac{1}{1 + \operatorname{Cos}[x]} (1 + i) e^x \operatorname{Cos}\left[\frac{x}{2}\right] \\ \left( (1 + i) \operatorname{Cos}\left[\frac{x}{2}\right] \operatorname{Hypergeometric2F1}[-i, 1, 1 - i, -e^{ix}] - e^{ix} \operatorname{Cos}\left[\frac{x}{2}\right] \operatorname{Hypergeometric2F1}[1, 1 - i, 2 - i, -e^{ix}] - (1 - i) \operatorname{Sin}\left[\frac{x}{2}\right] \right)$$

**Problem 553: Result more than twice size of optimal antiderivative.**

$$\int \frac{e^x}{1 - \operatorname{Cos}[x]} dx$$

Optimal (type 5, 26 leaves, 2 steps):

$$(-1 + i) e^{(1+i)x} \text{Hypergeometric2F1}\left[1 - i, 2, 2 - i, e^{ix}\right]$$

Result (type 5, 84 leaves):

$$\frac{1}{-1 + \text{Cos}[x]} (1 + i) e^x \text{Sin}\left[\frac{x}{2}\right] \left( (1 - i) \text{Cos}\left[\frac{x}{2}\right] + (1 + i) \text{Hypergeometric2F1}\left[-i, 1, 1 - i, e^{ix}\right] \text{Sin}\left[\frac{x}{2}\right] + e^{ix} \text{Hypergeometric2F1}\left[1, 1 - i, 2 - i, e^{ix}\right] \text{Sin}\left[\frac{x}{2}\right] \right)$$

**Problem 554: Result more than twice size of optimal antiderivative.**

$$\int \frac{e^x}{1 + \text{Sin}[x]} dx$$

Optimal (type 5, 30 leaves, 2 steps):

$$(-1 + i) e^{(1+i)x} \text{Hypergeometric2F1}\left[1 + i, 2, 2 + i, -i e^{-ix}\right]$$

Result (type 5, 61 leaves):

$$\frac{2 e^x \text{Sin}\left[\frac{x}{2}\right]}{\text{Cos}\left[\frac{x}{2}\right] + \text{Sin}\left[\frac{x}{2}\right]} - (1 - i) \left(1 - (1 - i) \text{Hypergeometric2F1}\left[-i, 1, 1 - i, i \text{Cos}[x] - \text{Sin}[x]\right]\right) (\text{Cosh}[x] + \text{Sinh}[x])$$

**Problem 555: Result more than twice size of optimal antiderivative.**

$$\int \frac{e^x}{1 - \text{Sin}[x]} dx$$

Optimal (type 5, 30 leaves, 2 steps):

$$(1 + i) e^{(1+i)x} \text{Hypergeometric2F1}\left[1 - i, 2, 2 - i, -i e^{ix}\right]$$

Result (type 5, 61 leaves):

$$\frac{2 e^x \text{Sin}\left[\frac{x}{2}\right]}{\text{Cos}\left[\frac{x}{2}\right] - \text{Sin}\left[\frac{x}{2}\right]} + (1 + i) \left(1 - (1 + i) \text{Hypergeometric2F1}\left[-i, 1, 1 - i, -i \text{Cos}[x] + \text{Sin}[x]\right]\right) (\text{Cosh}[x] + \text{Sinh}[x])$$

**Problem 557: Result more than twice size of optimal antiderivative.**

$$\int \frac{e^x (1 + \text{Sin}[x])}{1 - \text{Cos}[x]} dx$$

Optimal (type 5, 41 leaves, 7 steps):

$$(-2 + 2i) e^{(1+i)x} \text{Hypergeometric2F1}[1 - i, 2, 2 - i, e^{ix}] + \frac{e^x \sin[x]}{1 - \cos[x]}$$

Result (type 5, 100 leaves):

$$\left( 2 e^x \sin\left[\frac{x}{2}\right] \left( \cos\left[\frac{x}{2}\right] + 2i \text{Hypergeometric2F1}[-i, 1, 1 - i, e^{ix}] \sin\left[\frac{x}{2}\right] + (1 + i) e^{ix} \text{Hypergeometric2F1}[1, 1 - i, 2 - i, e^{ix}] \sin\left[\frac{x}{2}\right] \right) \right. \\ \left. (1 + \sin[x]) \right) / \left( (-1 + \cos[x]) \left( \cos\left[\frac{x}{2}\right] + \sin\left[\frac{x}{2}\right] \right)^2 \right)$$

**Problem 559: Result more than twice size of optimal antiderivative.**

$$\int \frac{e^x (1 - \sin[x])}{1 + \cos[x]} dx$$

Optimal (type 5, 42 leaves, 7 steps):

$$(2 - 2i) e^{(1+i)x} \text{Hypergeometric2F1}[1 - i, 2, 2 - i, -e^{ix}] - \frac{e^x \sin[x]}{1 + \cos[x]}$$

Result (type 5, 87 leaves):

$$-\frac{1}{1 + \cos[x]} \\ 2 e^x \cos\left[\frac{x}{2}\right] \left( 2i \cos\left[\frac{x}{2}\right] \text{Hypergeometric2F1}[-i, 1, 1 - i, -e^{ix}] - (1 + i) e^{ix} \cos\left[\frac{x}{2}\right] \text{Hypergeometric2F1}[1, 1 - i, 2 - i, -e^{ix}] - \sin\left[\frac{x}{2}\right] \right)$$

**Problem 574: Result more than twice size of optimal antiderivative.**

$$\int \text{sech}[x] dx$$

Optimal (type 3, 3 leaves, 1 step):

$$\text{ArcTan}[\text{Sinh}[x]]$$

Result (type 3, 9 leaves):

$$2 \text{ArcTan}\left[\text{Tanh}\left[\frac{x}{2}\right]\right]$$

**Problem 575: Result more than twice size of optimal antiderivative.**

$$\int \text{csch}[x] dx$$

Optimal (type 3, 5 leaves, 1 step):

$$-\text{ArcTanh}[\text{Cosh}[x]]$$

Result (type 3, 17 leaves):

$$-\text{Log}\left[\text{Cosh}\left[\frac{x}{2}\right]\right] + \text{Log}\left[\text{Sinh}\left[\frac{x}{2}\right]\right]$$

**Problem 579: Result more than twice size of optimal antiderivative.**

$$\int \text{Csch}[x]^3 dx$$

Optimal (type 3, 16 leaves, 2 steps):

$$\frac{1}{2} \text{ArcTanh}[\text{Cosh}[x]] - \frac{1}{2} \text{Coth}[x] \text{Csch}[x]$$

Result (type 3, 47 leaves):

$$-\frac{1}{8} \text{Csch}\left[\frac{x}{2}\right]^2 + \frac{1}{2} \text{Log}\left[\text{Cosh}\left[\frac{x}{2}\right]\right] - \frac{1}{2} \text{Log}\left[\text{Sinh}\left[\frac{x}{2}\right]\right] - \frac{1}{8} \text{Sech}\left[\frac{x}{2}\right]^2$$

**Problem 592: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{\text{Cosh}[x] (-\text{Cosh}[2x] + \text{Tanh}[x])}{\sqrt{\text{Sinh}[2x]} (\text{Sinh}[x]^2 + \text{Sinh}[2x])} dx$$

Optimal (type 3, 69 leaves, 8 steps):

$$\sqrt{2} \text{ArcTan}\left[\frac{\text{Sech}[x] \sqrt{\text{Cosh}[x] \text{Sinh}[x]}}{\sqrt{\text{Sinh}[2x]}}\right] + \frac{1}{6} \text{ArcTan}\left[\frac{\text{Sinh}[x]}{\sqrt{\text{Sinh}[2x]}}\right] - \frac{1}{3} \sqrt{2} \text{ArcTanh}\left[\frac{\text{Sech}[x] \sqrt{\text{Cosh}[x] \text{Sinh}[x]}}{\sqrt{\text{Sinh}[2x]}}\right] + \frac{\text{Cosh}[x]}{\sqrt{\text{Sinh}[2x]}}$$

Result (type 4, 487 leaves):

$$\begin{aligned}
& - \frac{\text{Coth}[x] \sqrt{\text{Sinh}[2x]} (-\text{Cosh}[2x] + \text{Tanh}[x])}{\text{Cosh}[x] + \text{Cosh}[3x] - 2\text{Sinh}[x]} + \\
& \frac{1}{2(\text{Cosh}[x] + \text{Cosh}[3x] - 2\text{Sinh}[x])} \text{Cosh}[x] \left( - \left( \left( 6(-1)^{1/4} \sqrt{1 + \text{Coth}\left[\frac{x}{2}\right]^2} \left( \text{EllipticF}\left[\text{i ArcSinh}\left[\frac{(-1)^{1/4}}{\sqrt{\text{Tanh}\left[\frac{x}{2}\right]}\right]}, -1\right] - \right. \right. \right. \\
& \left. \left. \left. \text{EllipticPi}\left[-(-1)^{1/6}, \text{i ArcSinh}\left[\frac{(-1)^{1/4}}{\sqrt{\text{Tanh}\left[\frac{x}{2}\right]}\right]}, -1\right] - \text{EllipticPi}\left[-(-1)^{5/6}, \text{i ArcSinh}\left[\frac{(-1)^{1/4}}{\sqrt{\text{Tanh}\left[\frac{x}{2}\right]}\right]}, -1\right] \right) \right) \right. \\
& \left. \left. \sqrt{\text{Sinh}[2x]} \sqrt{\text{Tanh}\left[\frac{x}{2}\right]} \sqrt{\text{Tanh}\left[\frac{x}{2}\right] + \text{Tanh}\left[\frac{x}{2}\right]^3} \right) / \left( (1 + \text{Cosh}[x]) \sqrt{\frac{\text{Sinh}[2x]}{(1 + \text{Cosh}[x])^2}} (1 + \text{Tanh}\left[\frac{x}{2}\right]^2) \right) \right) + \\
& \left( 16(-1)^{5/12} \left( (3 - 3\text{i}\sqrt{3}) \text{EllipticPi}\left[-\text{i}, \text{i ArcSinh}\left[(-1)^{1/4} \sqrt{\text{Tanh}\left[\frac{x}{2}\right]}\right]}, -1\right] + 2(-1 + (-1)^{1/3}) \right. \\
& \left. \text{EllipticPi}\left[\text{i}, \text{ArcSin}\left[(-1)^{3/4} \sqrt{\text{Tanh}\left[\frac{x}{2}\right]}\right]}, -1\right] + \text{i}(\text{i} + \sqrt{3}) \text{EllipticPi}\left[-(-1)^{1/6}, \text{i ArcSinh}\left[(-1)^{1/4} \sqrt{\text{Tanh}\left[\frac{x}{2}\right]}\right]}, -1\right] + \right. \\
& \left. 2(-1 + (-1)^{1/3}) \text{EllipticPi}\left[-(-1)^{5/6}, \text{i ArcSinh}\left[(-1)^{1/4} \sqrt{\text{Tanh}\left[\frac{x}{2}\right]}\right]}, -1\right] \right) \text{Sinh}[2x]^{3/2} \sqrt{\text{Tanh}\left[\frac{x}{2}\right] + \text{Tanh}\left[\frac{x}{2}\right]^3} \right) / \\
& \left( 3(-\text{i} + \sqrt{3}) (1 + \text{Cosh}[x])^3 \left( \frac{\text{Sinh}[2x]}{(1 + \text{Cosh}[x])^2} \right)^{3/2} \sqrt{\text{Tanh}\left[\frac{x}{2}\right]} \sqrt{1 + \text{Tanh}\left[\frac{x}{2}\right]^2} \right) \right) (-\text{Cosh}[2x] + \text{Tanh}[x])
\end{aligned}$$

**Problem 601: Result more than twice size of optimal antiderivative.**

$$\int e^{-2x} \text{Sech}[x]^4 dx$$

Optimal (type 3, 13 leaves, 3 steps):

$$-\frac{8}{3(1+e^{2x})^3}$$

Result (type 3, 32 leaves):

$$\frac{8e^{2x}(3+3e^{2x}+e^{4x})}{3(1+e^{2x})^3}$$

**Problem 622: Result more than twice size of optimal antiderivative.**

$$\int \frac{1}{x\sqrt{a^2 + \text{Log}[x]^2}} dx$$

Optimal (type 3, 16 leaves, 3 steps):

$$\text{ArcTanh}\left[\frac{\text{Log}[x]}{\sqrt{a^2 + \text{Log}[x]^2}}\right]$$

Result (type 3, 46 leaves):

$$-\frac{1}{2} \text{Log}\left[1 - \frac{\text{Log}[x]}{\sqrt{a^2 + \text{Log}[x]^2}}\right] + \frac{1}{2} \text{Log}\left[1 + \frac{\text{Log}[x]}{\sqrt{a^2 + \text{Log}[x]^2}}\right]$$

**Problem 623: Result more than twice size of optimal antiderivative.**

$$\int \frac{1}{x\sqrt{-a^2 + \text{Log}[x]^2}} dx$$

Optimal (type 3, 18 leaves, 3 steps):

$$\text{ArcTanh}\left[\frac{\text{Log}[x]}{\sqrt{-a^2 + \text{Log}[x]^2}}\right]$$

Result (type 3, 50 leaves):

$$-\frac{1}{2} \text{Log}\left[1 - \frac{\text{Log}[x]}{\sqrt{-a^2 + \text{Log}[x]^2}}\right] + \frac{1}{2} \text{Log}\left[1 + \frac{\text{Log}[x]}{\sqrt{-a^2 + \text{Log}[x]^2}}\right]$$

**Problem 627:** Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{x \operatorname{Log}[x] \sqrt{-a^2 + \operatorname{Log}[x]^2}} dx$$

Optimal (type 3, 23 leaves, 4 steps):

$$\frac{\operatorname{ArcTan}\left[\frac{\sqrt{-a^2 + \operatorname{Log}[x]^2}}{a}\right]}{a}$$

Result (type 3, 38 leaves):

$$-\frac{i \operatorname{Log}\left[-\frac{2 i a}{\operatorname{Log}[x]} + \frac{2 \sqrt{-a^2 + \operatorname{Log}[x]^2}}{\operatorname{Log}[x]}\right]}{a}$$

**Problem 689:** Result more than twice size of optimal antiderivative.

$$\int \frac{x^6 \operatorname{ArcSec}[x]}{(-1 + x^2)^{5/2}} dx$$

Optimal (type 4, 175 leaves, 16 steps):

$$\frac{\sqrt{x^2} (2 - 3 x^2)}{6 (-1 + x^2)} - \frac{13}{6} \operatorname{ArcCoth}\left[\sqrt{x^2}\right] - \frac{5 x^3 \operatorname{ArcSec}[x]}{6 (-1 + x^2)^{3/2}} + \frac{x^5 \operatorname{ArcSec}[x]}{2 (-1 + x^2)^{3/2}} - \frac{5 x \operatorname{ArcSec}[x]}{2 \sqrt{-1 + x^2}} -$$

$$\frac{5 i \sqrt{x^2} \operatorname{ArcSec}[x] \operatorname{ArcTan}\left[e^{i \operatorname{ArcSec}[x]}\right]}{x} + \frac{5 i \sqrt{x^2} \operatorname{PolyLog}\left[2, -i e^{i \operatorname{ArcSec}[x]}\right]}{2 x} - \frac{5 i \sqrt{x^2} \operatorname{PolyLog}\left[2, i e^{i \operatorname{ArcSec}[x]}\right]}{2 x}$$

Result (type 4, 383 leaves):



$$\begin{aligned}
& - \frac{1}{96 (-1+x^2)^{3/2}} x^5 \left( 22 \operatorname{ArcSec}[x] + 40 \operatorname{ArcSec}[x] \cos[2 \operatorname{ArcSec}[x]] - 30 \operatorname{ArcSec}[x] \cos[4 \operatorname{ArcSec}[x]] - 30 \sqrt{1-\frac{1}{x^2}} \operatorname{ArcSec}[x] \log[1-i e^{i \operatorname{ArcSec}[x]}] + \right. \\
& 30 \sqrt{1-\frac{1}{x^2}} \operatorname{ArcSec}[x] \log[1+i e^{i \operatorname{ArcSec}[x]}] + 26 \sqrt{1-\frac{1}{x^2}} \log\left[\cos\left[\frac{\operatorname{ArcSec}[x]}{2}\right]\right] - 26 \sqrt{1-\frac{1}{x^2}} \log\left[\sin\left[\frac{\operatorname{ArcSec}[x]}{2}\right]\right] + 16 \sin[2 \operatorname{ArcSec}[x]] - \\
& 60 i \sqrt{1-\frac{1}{x^2}} \operatorname{PolyLog}\left[2, -i e^{i \operatorname{ArcSec}[x]}\right] \sin[2 \operatorname{ArcSec}[x]]^2 + 60 i \sqrt{1-\frac{1}{x^2}} \operatorname{PolyLog}\left[2, i e^{i \operatorname{ArcSec}[x]}\right] \sin[2 \operatorname{ArcSec}[x]]^2 - \\
& 15 \operatorname{ArcSec}[x] \log[1-i e^{i \operatorname{ArcSec}[x]}] \sin[3 \operatorname{ArcSec}[x]] + 15 \operatorname{ArcSec}[x] \log[1+i e^{i \operatorname{ArcSec}[x]}] \sin[3 \operatorname{ArcSec}[x]] + \\
& 13 \log\left[\cos\left[\frac{\operatorname{ArcSec}[x]}{2}\right]\right] \sin[3 \operatorname{ArcSec}[x]] - 13 \log\left[\sin\left[\frac{\operatorname{ArcSec}[x]}{2}\right]\right] \sin[3 \operatorname{ArcSec}[x]] - 4 \sin[4 \operatorname{ArcSec}[x]] + \\
& 15 \operatorname{ArcSec}[x] \log[1-i e^{i \operatorname{ArcSec}[x]}] \sin[5 \operatorname{ArcSec}[x]] - 15 \operatorname{ArcSec}[x] \log[1+i e^{i \operatorname{ArcSec}[x]}] \sin[5 \operatorname{ArcSec}[x]] - \\
& \left. 13 \log\left[\cos\left[\frac{\operatorname{ArcSec}[x]}{2}\right]\right] \sin[5 \operatorname{ArcSec}[x]] + 13 \log\left[\sin\left[\frac{\operatorname{ArcSec}[x]}{2}\right]\right] \sin[5 \operatorname{ArcSec}[x]] \right)
\end{aligned}$$

**Problem 698: Result more than twice size of optimal antiderivative.**

$$\int -\frac{\operatorname{ArcTan}[a-x]}{a+x} dx$$

Optimal (type 4, 122 leaves, 5 steps):

$$\begin{aligned}
& \operatorname{ArcTan}[a-x] \log\left[\frac{2}{1-i(a-x)}\right] - \operatorname{ArcTan}[a-x] \log\left[-\frac{2(a+x)}{(i-2a)(1-i(a-x))}\right] - \\
& \frac{1}{2} i \operatorname{PolyLog}\left[2, 1-\frac{2}{1-i(a-x)}\right] + \frac{1}{2} i \operatorname{PolyLog}\left[2, 1+\frac{2(a+x)}{(i-2a)(1-i(a-x))}\right]
\end{aligned}$$

Result (type 4, 256 leaves):

$$\begin{aligned}
& -\text{ArcTan}[a-x] \left( \frac{1}{2} \text{Log}[1+a^2-2ax+x^2] + \text{Log}[-\text{Sin}[\text{ArcTan}[2a] - \text{ArcTan}[a-x]]] \right) + \\
& \frac{1}{2} \left( \frac{1}{4} i (\pi - 2 \text{ArcTan}[a-x])^2 + i (\text{ArcTan}[2a] - \text{ArcTan}[a-x])^2 - \right. \\
& (\pi - 2 \text{ArcTan}[a-x]) \text{Log}[1 + e^{-2i \text{ArcTan}[a-x]}] - 2 (-\text{ArcTan}[2a] + \text{ArcTan}[a-x]) \text{Log}[1 - e^{2i (-\text{ArcTan}[2a] + \text{ArcTan}[a-x])}] + \\
& (\pi - 2 \text{ArcTan}[a-x]) \text{Log}\left[\frac{2}{\sqrt{1+(a-x)^2}}\right] - 2 (\text{ArcTan}[2a] - \text{ArcTan}[a-x]) \text{Log}[-2 \text{Sin}[\text{ArcTan}[2a] - \text{ArcTan}[a-x]]] + \\
& \left. i \text{PolyLog}[2, -e^{-2i \text{ArcTan}[a-x]}] + i \text{PolyLog}[2, e^{2i (-\text{ArcTan}[2a] + \text{ArcTan}[a-x])}] \right)
\end{aligned}$$

**Problem 703: Result unnecessarily involves imaginary or complex numbers.**

$$\int \text{ArcSin}[\text{Sinh}[x]] \text{Sech}[x]^4 dx$$

Optimal (type 3, 49 leaves, 5 steps):

$$-\frac{2}{3} \text{ArcSin}\left[\frac{\text{Cosh}[x]}{\sqrt{2}}\right] + \frac{1}{6} \text{Sech}[x] \sqrt{1 - \text{Sinh}[x]^2} + \text{ArcSin}[\text{Sinh}[x]] \text{Tanh}[x] - \frac{1}{3} \text{ArcSin}[\text{Sinh}[x]] \text{Tanh}[x]^3$$

Result (type 3, 66 leaves):

$$\frac{1}{12} \left( 8 i \text{Log}[i \sqrt{2} \text{Cosh}[x] + \sqrt{3 - \text{Cosh}[2x]}] + \sqrt{6 - 2 \text{Cosh}[2x]} \text{Sech}[x] + 4 \text{ArcSin}[\text{Sinh}[x]] (2 + \text{Cosh}[2x]) \text{Sech}[x]^2 \text{Tanh}[x] \right)$$

**Problem 704: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \text{ArcCot}[\text{Cosh}[x]] \text{Coth}[x] \text{Csch}[x]^3 dx$$

Optimal (type 3, 36 leaves, 6 steps):

$$\frac{\text{ArcTanh}\left[\frac{\text{Tanh}[x]}{\sqrt{2}}\right]}{6\sqrt{2}} + \frac{\text{Coth}[x]}{6} - \frac{1}{3} \text{ArcCot}[\text{Cosh}[x]] \text{Csch}[x]^3$$

Result (type 3, 144 leaves):

$$\begin{aligned}
& \frac{1}{48} \text{Csch}[x]^3 \left( -16 \text{ArcCot}[\text{Cosh}[x]] - 2 \text{Cosh}[x] + 2 \text{Cosh}[3x] - 3 i \sqrt{2} \text{ArcTan}\left[1 - i \sqrt{2} \text{Tanh}\left[\frac{x}{2}\right]\right] \text{Sinh}[x] + \right. \\
& \left. 3 i \sqrt{2} \text{ArcTan}\left[1 + i \sqrt{2} \text{Tanh}\left[\frac{x}{2}\right]\right] \text{Sinh}[x] + i \sqrt{2} \text{ArcTan}\left[1 - i \sqrt{2} \text{Tanh}\left[\frac{x}{2}\right]\right] \text{Sinh}[3x] - i \sqrt{2} \text{ArcTan}\left[1 + i \sqrt{2} \text{Tanh}\left[\frac{x}{2}\right]\right] \text{Sinh}[3x] \right)
\end{aligned}$$

### Problem 705: Result more than twice size of optimal antiderivative.

$$\int e^x \operatorname{ArcSin}[\operatorname{Tanh}[x]] \, dx$$

Optimal (type 3, 28 leaves, 5 steps):

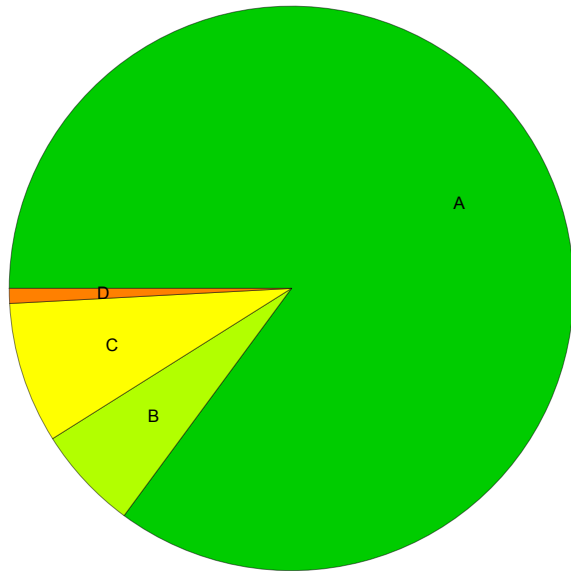
$$e^x \operatorname{ArcSin}[\operatorname{Tanh}[x]] - \operatorname{Cosh}[x] \operatorname{Log}[1 + e^{2x}] \sqrt{\operatorname{Sech}[x]^2}$$

Result (type 3, 64 leaves):

$$e^x \operatorname{ArcSin}\left[\frac{-1 + e^{2x}}{1 + e^{2x}}\right] - e^{-x} \sqrt{\frac{e^{2x}}{(1 + e^{2x})^2}} (1 + e^{2x}) \operatorname{Log}[1 + e^{2x}]$$

## Summary of Integration Test Results

705 integration problems



A - 600 optimal antiderivatives

B - 42 more than twice size of optimal antiderivatives

C - 57 unnecessarily complex antiderivatives

D - 6 unable to integrate problems

E - 0 integration timeouts